ASTR 5590 - Chapter 7a

Magnetic fields are everywhere in the Universe, & charged particles react to & affect them

* How are their dynamics affected by B?

Lorentz force, $\vec{F} = q\,\vec{v} \times \vec{B}$

$$\vec{F} = \frac{d}{dt}\vec{p} = \frac{d}{dt}(\gamma m_0 \vec{v}) = ze(\vec{v} \times \vec{B})$$

Left side: $= m_0\left(\frac{d\gamma}{dt}\vec{v} + \gamma\frac{d\vec{v}}{dt}\right)$

$-\frac{1}{c^2}\left(\vec{v}\cdot\frac{d\vec{v}}{dt} + \frac{d\vec{v}}{dt}\cdot\vec{v}\right)$

$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$, $\frac{d\gamma}{dt} = -\frac{1}{2}\left(1 - \frac{v^2}{c^2}\right)^{-3/2}\left(-\frac{2}{c^2}\right)\vec{v}\cdot\frac{d\vec{v}}{dt}$

($\vec{v}\cdot\vec{v}$)

$$= m_0\left(\gamma^3\vec{v}\,\frac{\vec{v}\cdot\vec{a}}{c^2} + \gamma\frac{d\vec{v}}{dt}\right)$$

$\vec{a} \perp \vec{v}$ (cross product), so

$$\gamma m_0 \frac{d\vec{v}}{dt} = ze(\vec{v} \times \vec{B})$$

$a_{\parallel}$ (to $\vec{B}$) is 0, $a_{\perp}$ depends on angle

* Show fig 7.1

$v_{\parallel}$ is const. → no accel.

recall $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta\,\vec{n}$

$$\gamma m_0 \frac{d\vec{v}}{dt} = ze|\vec{v}||\vec{B}|\sin\theta\,(\hat{i}_v \times \hat{i}_B)$$

$$v_\perp = |\vec{v}|\sin\theta$$

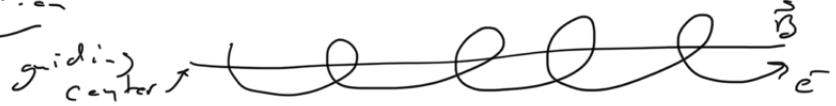
Circular motion $\perp$ to $\vec{B}$, so

$$a_{circ} = \frac{v_\perp^2}{r} = \left|\frac{d\vec{v}}{dt}\right| = \frac{ze|\vec{v}||\vec{B}|\sin\theta}{\gamma m_0}$$

$$\text{or} \quad r_{circ} = \frac{\gamma m_0 |\vec{v}|\sin\theta}{ze|\vec{B}|}$$

↖ gyroradius or cyclotron rad.

Spiral motion



$$\omega_g = \frac{v_\perp}{r} = \frac{ze|\vec{B}|}{\gamma m_0}$$

$$\nu_g = 2.8\ \text{MHz}\left(\frac{B}{1\,G}\right)$$

Can use $p = \gamma m_0 |\vec{v}|$ & rewrite

$$r = \left(\frac{pc}{ze}\right)\frac{\sin\theta}{|\vec{B}|\,c}$$

↳ "rigidity" or "magnetic rigidity"

has units of E/e, or volts