ASTR 5590 - Chapter 7b

★ What if $\vec{B}$ varies w/time (according to the particle as it travels)?

- assume $\left(\frac{\Delta B}{B}\right)_{orbit}$ is small
- assume non-rel.

$\perp$ to $\vec{B}$, particle travels in a circle, equivalent to current around a circular wire

$$I\text{ (current)} = \frac{\text{charge}}{\text{time}} = \frac{ze}{2\pi r / v_\perp}$$

Magnetic moment of loop is force it can exert on electric currents, defined as $\mu = IA$ ← area of loop $= \pi r^2$

$$\mu = \frac{ze v_\perp}{2} r, \quad r = \frac{\gamma m_0 v_\perp}{ze|\vec{B}|}$$

so in non-rel. case, $\mu = \frac{m_0 v_\perp^2}{2} \frac{1}{B}$

$KE \rightarrow W_\perp$ so $\mu = \frac{W_\perp}{B}$

Now, if B changes by small ΔB, that induces an electromotive force $\mathcal{E}$

in the loop that accel. the particle

$$\mathcal{E} = A\frac{dB}{dt}$$

Consider our change ΔB occurs over 1 orbit, so $\Delta T = \frac{2\pi r}{v_\perp}$

Work done by $\mathcal{E}$ is $q\mathcal{E} = \Delta W_\perp$ (change in KE)

$$\Delta W_\perp = \frac{ze\pi r^2 \Delta B}{2\pi r / v_\perp} = \frac{zerv_\perp}{2}\Delta B$$

but again, $r = \frac{m_0 v_\perp}{zeB}$ so $\Delta W_\perp = \underbrace{\frac{1}{2}m_0 v_\perp^2}_{W_\perp}\frac{\Delta B}{B}$

or, the frac. change in B is = frac. change in $W_\perp$

HERE →

Since $\mu = \frac{W_\perp}{B}$, $\Delta\mu = \Delta\left(\frac{W_\perp}{B}\right)$

$$\Delta\mu = \frac{B\Delta W_\perp - W_\perp \Delta B}{B^2} = \frac{\Delta W_\perp}{B} - \frac{W_\perp \Delta B}{B^2}$$

& $\frac{\Delta B}{B} = \frac{\Delta W_\perp}{W_\perp}$, $\Delta\mu = \frac{\Delta W_\perp}{B} - \frac{\cancel{W_\perp}}{B}\frac{\Delta W_\perp}{\cancel{W_\perp}}$

$$= 0$$

Mag. moment is <u>invariant</u>, or equivalently the KE/B is invariant

Since $W_\perp = \frac{1}{2}m_0 v_\perp^2 = \frac{p_\perp^2}{2m_0}$, also true

that $\Delta\left(\frac{p_\perp^2}{\ }\right) = 0$

Of course, $\vec{B}$ can't do work on the particle, so the total KE is also invariant

If $B\uparrow$, $p_\perp \uparrow$ & thus $p_\parallel \downarrow$ $\left(p^2 = p_\perp^2 + p_\parallel^2\right)$

Once $p \to p_\perp$, forward motion of particle stops & gets reflected back

Substitution to $\Delta\left(\frac{p_\perp^2}{B}\right) \to \Delta(Br^2) = 0$ also, so area $\propto B$: particle tied to B flux lines

☆ Fig. 7.2 & van Allen belts

<u>Relativistic versions</u>

$$\Delta(Br^2) = 0 \qquad r = \frac{\gamma m_0 v_\perp}{zeB}$$

$$\Delta(p_\perp^2/B) = 0 \qquad p_\perp = \gamma m_0 v_\perp$$

$$\Delta(\gamma\mu) = 0 \qquad \mu = \gamma m_0 v_\perp^2/2B$$