# ASTR 5590 - Chapter 7c

Scattering particles when B is not slowly varying

Pitch angles θ will generally be assumed to be uniformly dist., as can be expected if irregularities in B are random, so scatterings are random

1) if gyroradius smaller than fluctuations adiabatic invariants hold & pitch angles vary based on B randomly ↑ ↓

2) if $r_g$ larger than fluctuations, then just feel avg. B, so also no scatter

→ to scatter, need fluctuations on <u>same</u> <u>scale</u> as $r_g$

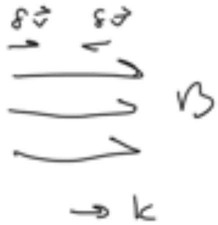
★ Fig. 7.5

The rigidity of CRs ~ 1 GeV/nucleon correspond to fluctuations in the solar wind — why they turn over in "Swordy" plot

Scattering by Alfvén & hydromagnetic waves

- in equilibrium, plasma is unperturbed w/ $\rho_0, \vec{B}_0, P_0$ = const, $\vec{v}_0 = 0$ = const.

Small perturbations take on wave-like form, so $\delta\rho, \delta P, \delta\vec{B}, \delta\vec{v} \propto e^{i(\vec{k}x - \omega t)}$




longitudinal $\delta\vec{v} \parallel \vec{k}$ $\omega^2 = k^2 c_s^2$ sound waves


transverse $\delta\vec{v} \perp \vec{k}, \vec{B}_0$ $\omega^2 = k^2 v_A^2$ Alfvén waves

$$V_A = \frac{B_0}{(\mu_0 \rho)^{1/2}}$$

$\rho = N_p m_p$


gas & B compressed $\delta\vec{v} \parallel \vec{k}$ longitudinal $\omega^2 = k^2(c_s^2 + V_A^2)$

→ These can scatter pitch angles & thru resonances gain energy & grow instabilities, even if none are present to begin with

HE [illegible] have mean free path

ITL particles which ...

$$\lambda_{sc} \approx r_g/\varphi^2 \text{ (random walk)}, \text{ where } \varphi = \frac{B_1}{B_0}$$

$B_0 \to$ avg. field, $B_1 \to$ random extra comp.

$\to$ on avg, pitch angle changes by $\varphi$ b/c that's amount $\vec{B}$ changed over $\sim 1$ orbit

For the pitch angle to change significantly, say by 1 radian, need $N^{1/2}\varphi = 1$ rad. where $N = \#$ scatterings

So $\lambda_{sc} \approx N\lambda = N r_g = r_g \varphi^{-2}$

↑ of fluctuations

Particles in resonance will lose momentum to the waves & grow those instabilities

Growth rate in energy density goes as

$$U = U_0 e^{\Gamma t}$$

$$w/\ \Gamma = \omega_g \frac{N(\geq E)}{N_p}\left(\frac{|v|}{v_A} - 1\right)$$

Instability grows until $v \sim v_A$, with

$v >> v_A$ to start

So, if CRs stream along B lines, create waves that take away momentum from CRs until they slow to $v_A$ → non-rel. in ISM

However, only true if medium is fully ionized → neutral-ion collisions can damp Alfvén waves, so growth can be arrested

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All very complicated, but effect is that CRs perform a random walk & "diffuse" from their source

In general, CRs will be injected into a volume with rate $Q(E,t)\,dV$, they lose energy w/rate $b(E)$ in $dV$, & they diffuse away from volume @ some rate following the gradient of the particle dist. $N(E)$. Therefore this dist. changes over time following the diffusion-loss eqn.:

$$\frac{dN(E)}{dt} = \frac{d}{dE}\left[b(E)N(E)\right]$$

$$\frac{\quad}{dt} = \frac{\quad}{dE}\left[\ \ldots\ \right]$$

$$+ \; Q(E, t)$$

$$+ \; D \, \nabla^2 N(E)$$

↖ diffusion coeff.