ASTR 5590 - Chapter 11c

Electrical Conductivity in plasma

If e^- move together, same as current

Are plasmas insulators or conductors?

Current density $\vec{J} = \sigma \vec{E}$

$$\vec{F} = -e\vec{E}, \quad \vec{a} = -\frac{e}{m_e}\vec{E}$$

thru dark arts, $\vec{a} \sim \frac{\vec{v}}{t}$, so $\vec{v} = -\frac{e}{m_e}\vec{E}t$

Current can only persist as long as e^- out of equilibrium, so $t = \tau_s(e,e)$

$$\langle \vec{v} \rangle \approx -\frac{e}{m_e}\vec{E}\,\tau_s(e,e)$$

Current density is charge thru some area in some time, so can also write

as $$\vec{J} = n_e \langle \vec{v} \rangle (-e) \approx \underbrace{\frac{e^2}{m_e} n_e \tau_s(e,e)}_{\sigma \text{ electrical conductivity}} \vec{E}$$

$$\sigma \approx \frac{e^2}{m_e} n_e \frac{2\pi \epsilon_0^2 m_e^2}{e^4 n_{e,p} \ln\Lambda} \left(\frac{2kT_e}{m_e}\right)^{3/2}$$

$$\sigma \approx \frac{2\pi \epsilon_0^2 m_e}{e^2 \ln\Lambda} \left(\frac{2kT_e}{m_e}\right)^{3/2}$$

→ doesn't depend on n_e (if $n_e \uparrow$, have more charge, but also more collisions [timescale shorter], so effects cancel)

$T \sim 10^6 K$, temp of ISM, conductivity is same as copper

→ charges travel effectively, can't get buildup of charge, large scales looks neutral!

Magnetic Flux Freezing

☆ Fig. 11.2

b/c σ is high, let's assume it's ∞

Following the figure, electromotive force induced in the circuit is simply change in B, so $\mathcal{E} = -\frac{d\Phi}{dt}$

where Φ is magnetic flux thru loop

Φ consists of part created by loop, + part due to all external currents

$$\Phi = \Phi_i + \Phi_{ex}, \quad \Phi_i = L I$$

(L ↑ inductance, I ↑ current)

If external currents change, also induce change in current in loop

$$-\frac{d\Phi}{dt} = -L\frac{dI}{dt} - \frac{d\Phi_{ex}}{dt} = RI$$

But $R = 0$ b/c $\sigma \to \infty$, or

$$\Phi_i + \Phi_{ex} = \text{const.}$$

→ If field changes, current loop compensates in opposite sense

What if loop changes shape? ☒ Fig. 11.3

In this case, $\mathcal{E} = E\ell$

where $|\vec{E}| = |\vec{v} \times \vec{B}| = vB_1$

Also, $\mathcal{E} = -\frac{d\Phi}{dt} = vB_1\ell$ or $d\Phi = -vB_1\ell dt$

But, change in mag. flux is $\Delta\Phi = B_1\Delta A$

or $d\Phi = B_1\ell dx$, where $v = \frac{dx}{dt}$

$= B_1\ell v\,dt$

→ 2 effects again cancel!

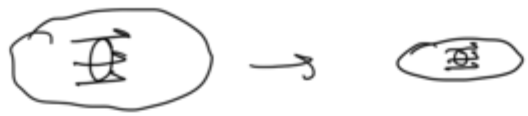
Equivalent of saying $\int_S \vec{B}\cdot d\vec{S} = \text{const.}$

Uniform field, $B\pi r^2 = \text{const.}$

or $B \propto r^{-2}$

If plasma compressed, $B\uparrow$ to compensate #

such that loop always has same
field lines → <u>B is frozen-in</u>



Examples

A) Star core collapses → NS, r↓ by orders of mag.
so B↑↑ ⇒ pulsar

B) Shocks compress ⇒ B↑

C) Stellar wind ⇒ galaxy rotation induces
spiral B fields (same w/acc. disk)