ASTR 5590 - Chapter 11d

Ideal Gas Review $\quad P = (\gamma - 1)\,\mathcal{E}$

A perfect gas relates P & $\mathcal{E}$ (E dens.) by the adiabatic index γ (not Lorentz factor)

☆ What is γ?

ratio of heat capacities @ const. P & const. V $\quad \gamma = \frac{c_p}{c_v}$ → E to raise unit mass $1\,\Delta T$ of temp

Ideal Gas → $PV^{\gamma} = \text{const.}$: defines an adiabat (no heat added to system)

→ $P \propto \rho^{\gamma}$

Like before w/ MHD waves, can have small perturbations $P = P_0 + \delta P$, $\frac{\delta P}{P} \ll 1$, expand to 1^{st} order, get wave eqn.s

Sound speed is $c_s = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_s}$ (Newton's law plus conservation mass)

or since $\frac{\partial P}{\partial \rho} = \gamma \rho^{\gamma - 1} = \gamma \frac{P}{\rho}$, $\boxed{c_s = \sqrt{\gamma \frac{P}{\rho}}}$

	Ideal Gas non-rel. gas	Radiation relativistic gas
γ	$5/3$	$4/3$
P	$\rho k T$	$\frac{4\sigma}{\ldots} T^4$

	$\overline{}\over \mu m_p$	$\frac{}{3}\frac{}{c}T$
$\mathcal{E}$	$\frac{3}{2}\frac{\rho k T}{\mu m_p}$	$\frac{4\sigma}{c}T^4$

$\sigma \rightarrow$ Stephan-Boltzmann const.

$= 5.67\times10^{-5}$ erg $/(\text{cm}^2 K^4 s)$

$\mu m_p \rightarrow$ mean mass per particle

$\mu = 1$ for neutral H

$= \frac{1}{2}$ for 100% ionized H

Here $\rightarrow$

Shock Waves

Fig. 11.6

In frame of shock (b), situation is in steady state: $\frac{\partial}{\partial t} = 0$

Either side of shock, quantities conserved:

mass | energy | momentum

Mass | Only change in x direction (y, z unaffected)

flux of particles thru an area

$\#_{\text{dens.}} = \text{flux} \cdot \frac{1}{v}$, @ shock

$\text{flux}_{\text{in}} = \text{flux}_{\text{out}}$, so $\boxed{\rho_1 v_1 = \rho_2 v_2}$

Energy) fluid dynamics: E flux thru a surface normal to $\vec{v}$ is $\rho\vec{v}\left(\frac{1}{2}v^2+w\right)$

$w \equiv$ enthalpy per unit mass

$= \mathcal{E}_m + PV$ ← vol. per unit mass

↑ internal energy per unit mass

→ conservation gives

$$\rho_1 v_1\left(\tfrac{1}{2}v_1^2 + w_1\right) = \rho_2 v_2\left(\tfrac{1}{2}v_2^2 + w_2\right)$$

$$\boxed{v_1\left(\tfrac{1}{2}\rho_1 v_1^2 + E_1 + P_1\right) = v_2\left(\tfrac{1}{2}\rho_2 v_2^2 + E_2 + P_2\right)}$$

Momentum) $\boxed{P_1 + \rho_1 v_1 = P_2 + \rho_2 v_2}$

- regular momentum mv, plus pressure (force per area)
  - → think of P as pushing gas

These 3 called Shock Jump Conditions

Assuming adiabatic conditions on both sides (but different adiabats), rewrite energy conservation using

$$E/V = \left(\frac{1}{\gamma-1}\right) P$$

$$\boxed{v_1^2 + \frac{2\gamma}{\gamma-1}\frac{P_1}{\rho_1} = v_2^2 + \frac{2\gamma}{\gamma-1}\frac{P_2}{\rho_2}}$$

Can define Mach # $\Rightarrow$ $\frac{\text{velocity of shock}}{\text{sound speed in pre-shock gas}}$

$$M_1 = \frac{v_s}{c_{s,1}}, \quad M^2 = \frac{\rho_1}{\gamma P_1} v_s^2, \quad v_1 = |v_s|$$

From mass cons. & E cons.

$$\boxed{\frac{v_2}{v_1} = \frac{\rho_1}{\rho_2} = \frac{2}{\gamma+1}\frac{1}{M_1^2} + \frac{\gamma-1}{\gamma+1}}$$

From mom. cons. & E cons.

$$\boxed{\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - (\gamma-1)}{\gamma+1}}$$

Using Ideal Gas $P = \rho k T$

$$\boxed{\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 - (\gamma-1)][2+(\gamma-1)M_1^2]}{(\gamma+1)^2 M_1^2}}$$

$$M_1^2 \geq 1$$

Consider Strong Shocks, $M \gg 1$, above reduces to

$\gamma \to 5/3$

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1} M^2 \longrightarrow \frac{5}{4} M^2$$

$$\frac{\rho_2}{\rho_1} = \frac{\gamma+1}{\gamma-1} \longrightarrow 4$$

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)M_1^2}{(\gamma+1)^2} \longrightarrow \frac{5}{16} M_1^2$$