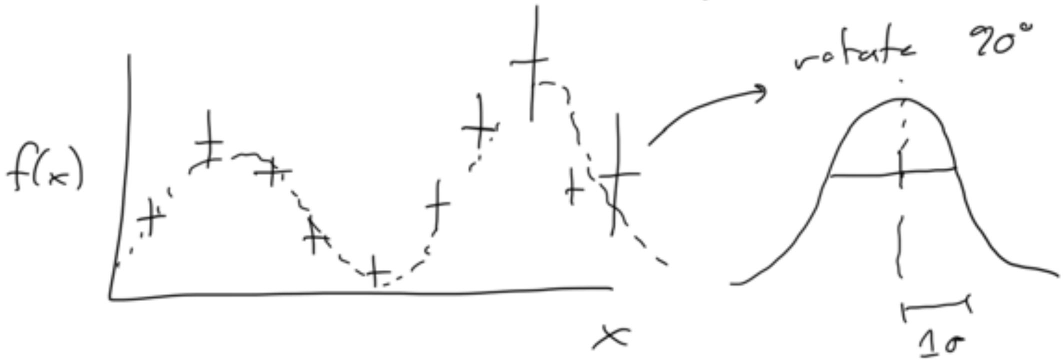
ASTR 5590 - Chi^2 Statistic

$\boxed{\text{Aside!}}$ Say you have a bunch of measurements that you want to fit a model to → how do you do it?



At each point, D_i, a given measurement @ that value of x is represented by the Gaussian dist. (a normal dist.)

where $g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$

"Error bars" ~~are~~ generally represent an estimate of the width of a Gaussian probability dist.

→ 68% chance true value w/in 1σ

95% chance " " w/in 2σ

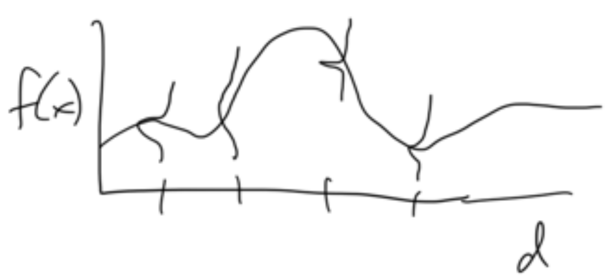
99.7% chance w/in 3σ

Found by integrating $f(x)$ from $-n\sigma$ to $+n\sigma$

$G(\mu + n\sigma) - G(\mu - n\sigma) = \mathrm{erf}\left(\frac{n}{\sqrt{2}}\right)$

In HEA, 90% uncertainty ranges are usually quoted : $\sigma\sqrt{2.706}$

χ^2 statistic is a function that allows one to find the best PARAMETERS of a model given data DRAWN from a Gaussian dist.



Data point at each d_i is randomly selected from $g(x)$ given σ_i (can vary)

If true, then best model will be the one that minimizes χ^2

$$\chi^2 = \sum_i \frac{(D_i - M_i)^2}{\sigma_i^2} = \sum_i \frac{(D_i - M_i)^2}{M_i}$$

If no uncertainty estimates on D_i, then 2^{nd} form can be used, but no way to weight points differently for quality

First form better, as it can accommodate more or less precise measurements

In HEA, we are able to count "events," photons or particles, so the proper dist. is the Poisson dist.

$$P(k \text{ events in interval}) = \frac{\lambda^k e^{-\lambda}}{k!}$$

where λ is the avg. # of events ($^{\text{real}}_{\#}$)

This is scary, BUT as k becomes large, a Poisson dist. converges to $g(x)$

What is large? No right answer, but ~25
→ uncertainty estimated from $\sqrt{C}$, $C \equiv$ counts
so $C = 25$ has $S/N \sim \sqrt{C} = 5$, or $\sim 5\sigma$
(really, just is when dist. pretty similar)

So, make sure your bins have $\gtrsim 25$ cts each, can use χ^2:

$$D_i = C$$

$$\sigma_i = \sqrt{C} \rightarrow \text{way to guess @ value of } \sigma \text{ (gaussian)}$$

True uncertainty is given by $\sqrt{M_i}$, but M_i is what we're trying to find

In practice, have $M(x_1, x_2, \ldots)$, vary x_1, x_2, etc, calc. χ^2, until a global minimum is found — this gives your "best-fit values" for x_1, x_2, etc

★ How do you estimate uncertainties for x_1, x_2, etc.?