ASTR 5590 - Chapter 8a

Synchrotron Radiation

What have we learned so far?

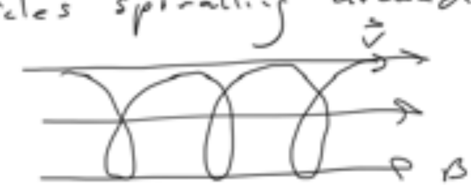
- E lost by particles thru Coulomb interactions
- radiation emitted thru ↗
- properties of a magnetized plasma
  - how particles move & how fields react to plasma

What's missing is radiation emitted by particles in a plasma

Considering bremsstrahlung energy losses, found

$$\left(\frac{dE}{dt}\right) = \frac{q^2\gamma^4}{6\pi\epsilon_0 c^3}\underbrace{\left(|a_\perp|^2 + \gamma^2 |a_\parallel|^2\right)}_{\text{relative to } \vec{v}}$$

For particles spiralling around $\vec{B}$ lines,



have a freq. $\nu_g = \frac{eB}{2\pi\gamma m_e} = 28\,\gamma^{-1}\ \text{GHz}\ \text{T}^{-1}$

$= 2.8\,\gamma^{-1}\ \text{MHz}\ \text{G}^{-1}$

& $a_\perp = \frac{evB\sin\alpha}{\gamma m_e}$, $a_\parallel = 0$

Why is $a_\parallel = 0$? What is α?

So, $-\left(\frac{dE}{dt}\right) = \frac{e^4 B}{\ldots}\,\frac{v^2}{c^2}\gamma^2\sin^2\alpha$

$$\left(\frac{dE}{dt}\right)_{rad} \quad 6\pi\epsilon_0 c m_e$$

using $c^2 = \frac{1}{\mu_0 \epsilon_0}$ & $U_B = \frac{B^2}{2\mu_0}$

can rewrite

$$-\left(\frac{dE}{dt}\right) = 2\underbrace{\left(\frac{e^4}{6\pi\epsilon_0^2 c^4 m_e^2}\right)}_{\sigma_T}\left(\frac{v}{c}\right)^2 c \frac{B^2}{2\mu_0} \gamma^2 \sin^2\alpha$$

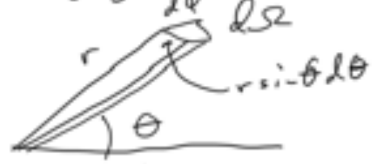
$$-\left(\frac{dE}{dt}\right) = 2\sigma_T c U_B \left(\frac{v}{c}\right)^2 \gamma^2 \sin^2\alpha$$

☆ In ultra-rel. limit, how can we simplify?

☆ What can we say about the pitch angles?

Typical E-loss given by averaging over α

$$\frac{\partial N}{\partial \alpha} = \text{const.}$$



Solid angle, so $r=1$ (unit sphere), pitch doesn't depend on φ, get $2\pi \sin\alpha\, d\alpha$

Avg.: $$\frac{\int_0^\pi -\left(\frac{dE}{dt}\right) 2\pi \sin\alpha\, d\alpha}{\int_0^\pi 2\pi\sin\alpha\, d\alpha \to 4\pi}$$

$$\hookrightarrow 2\sigma_T c U_B \gamma^2 \left(\frac{v}{c}\right)^2 \frac{1}{2}\int_0^\pi \sin^3\alpha\, d\alpha \rightarrow$$

$\int (1-\cos^2\alpha)\sin\alpha\, d\alpha$

$x = \cos\alpha$

$dx = -\sin\alpha\, d\alpha$

$\int_{-1}^{1}(1-x^2)\,dx$

$x - \frac{x^3}{3}$

$= 2 - \frac{2}{3}$

$$\boxed{-\left(\frac{dE}{dt}\right)_{avg} = \frac{4}{3}\sigma_T c U_B \left(\frac{v}{c}\right)^2 \gamma^2}$$

Consider Non-Rel. Case 1ST

Cyclotron Radiation

★ What does the spectrum look like?

w/e $\hat{\omega}$
v
w/s $\perp$ –

- linearly polarized when los $\perp$ B
- circularly polarized when los $\parallel$ B
- elliptically polarized in general case

As e^- become mildly rel., ν no longer simply ν_g → beaming effects factor in

Can decompose into sum of dipoles or harmonics ℓ:

$$\nu_\ell = \frac{\ell \; \nu_r}{\left(1 - \frac{v_\parallel}{c}\cos\theta\right)}$$

$\underline{\nu_r = \nu_g / \gamma}$

↖ Doppler shift

vel. to los θ of obs.

Total Power in ↑ harmonics goes as

$$\left(\frac{dE}{dt}\right)_{\ell+1} \Big/ \left(\frac{dE}{dt}\right)_\ell \approx \left(\frac{v}{c}\right)^2$$

so rad. falls w/ freq. Fig. 8.2

<u>This</u> is synchrotron radiation

In binaries, cyclotron features allow B to be estimated + sometimes field configuration based on polarization

---

<u>Relativistic Case : Synchrotron</u>

Have to consider how radiation gets transformed b/t ref. frames

★ Fig 8.5 $\boxed{\text{Assume } \alpha = 90°}$

In its frame, radiation is dist. [illegible]
as in the case for brem.

Converting : $\sin\phi = \frac{1}{\gamma}\frac{\sin\phi'}{1+\left(\frac{v}{c}\right)\cos\phi'}$

For $\gamma >> 1$, that term dominates &

$$\sin\phi \approx \phi \approx \pm\frac{1}{\gamma}$$

Emission beamed in direction of e^- travel

8.5b

Observed when $\vec{v}$ within $\pm\frac{1}{\gamma}$ of los

Freq. of emission is then Fourier trans.
of the pulse → duration much shorter
than orbit, so obs. ν ↑ than ν_g

8.5c

Duration of pulse is $< \frac{1}{\gamma}$ though
b/c 1^{st} photons emitted from farther
away than last photons of pulse

$$\Delta t = \underbrace{\frac{R-L}{c}}_{\text{last}} + \underbrace{\frac{L}{v}}_{1^{st}} - \frac{R}{c}$$

distances traveled

time delay of last pulse rel. to 1^{st}

$$\Delta t = \frac{L}{v}\left[\frac{v}{c}\left(\frac{R}{L}-1\right)+1-\frac{v}{c}\frac{R}{L}\right]$$

$$\Delta t = \frac{L}{v}\left[1-\frac{v}{c}\right]$$

Given that $\frac{L}{v} = \frac{r_s\theta}{v}$ ← from 8.5c

$$\approx \frac{1}{\cdot}\frac{1}{\cdot} = \frac{1}{\cdot}$$

$\omega_r \; \gamma \quad \omega_g$

rel. non-rel.

Also, $1 - \frac{v}{c} = \frac{1 - v^2/c^2}{1 + v/c} = \frac{1}{\gamma^2} \frac{1}{1 + v/c} \approx \frac{1}{2\gamma^2}$

(multiply top/bottom by)

So $\Delta t \approx \frac{1}{2\gamma^2 \omega_g}$, $\sim \frac{1}{\gamma^2}$ shorter than ω_g

Max. Fourier component should be this time,

$\nu \sim \Delta t^{-1} \sim \gamma^2 \nu_g$

↳ corresponds to freq. where most of radiation is emitted

In general, $\nu \sim \gamma^2 \nu_g \sin\alpha$

Proper critical freq $\boxed{\nu_c = \frac{3}{2} \gamma^2 \nu_g \sin\alpha}$