ASTR 5590 - Chapter 8b

Full treatment for synchrotron - results

Follow same procedure as w/ brems., do Fourier trans. to get spectrum ....

Total emissivity of a single e^- is

$$j(\omega) = \frac{3^{1/2}}{8\pi^2 \epsilon_0} \frac{B \sin\alpha}{c m_e} F(x)$$

$$F(x) = x \int_x^{\infty} K_{5/3}(z)\,dz$$

$$x = \frac{2\omega a}{3c\gamma^3}$$, $a \rightarrow$ radius of curvature of orbit

$K_\gamma \rightarrow$ modified Bessel func.

Can use approx for $F(x)$ @ large + small x

$$F(x) \approx \begin{cases} \dfrac{4\pi}{3^{1/2}\,\Gamma(\frac{1}{3})} \left(\dfrac{x}{2}\right)^{1/3} & x \ll 1 \\ \left(\dfrac{\pi}{2}\right)^{1/2} x^{1/2} \exp(-x) & x \gg 1 \end{cases}$$

$\uparrow \nu$, $j(\nu) \propto \nu^{1/2} e^{-\nu/\nu_c}$ Fig. 8.8

$\downarrow \nu$, $j(\nu) \propto \nu^{1/3}$

Non-thermal particle dist. are generally power-laws : $N(E)dE = \kappa E^{-p} dE$

$N(E)$ is # dens. of e^- in interval $E \rightarrow E + dE$

Recall that E lost by a single e^- is

$$-\left(\frac{dE}{dt}\right) = \frac{4}{3} \sigma_T c\, U_B \gamma^2$$

which corresponds to luminosity of e^-, so total luminosity of PL dist.:

$L_{syn} = \frac{4}{3} \sigma_T c U_B \int_1 N(\gamma) d\gamma$

where $N(\gamma)$ is # from $\gamma \to \gamma + d\gamma$

In terms of ν, we assume each e^- radiates @ $\nu \sim \nu_0 \approx \gamma^2 \nu_g = \left(\frac{E}{m_e c^2}\right)^2 \frac{eB}{2\pi m_e}$

so $\nu \to \nu + d\nu$ equates to $E \to E + dE$

& $J(\nu) d\nu = \left(-\frac{dE}{dt}\right) N(E) dE$

& since $\gamma \approx \left(\frac{\nu}{\nu_g}\right)^{1/2}$, $E = \gamma m_e c^2$,

$$dE = \frac{d}{d\nu} \gamma m_e c^2 = \frac{1}{2} m_e c^2 \frac{\nu^{-1/2}}{\nu_g^{1/2}} d\nu,$$

$$-\frac{dE}{dt} = \frac{4}{3} \sigma_T c \frac{B^2}{2\mu_0} \left(\frac{E}{m_e c^2}\right)^2$$

$$\boxed{J(\nu) \propto K B^{(p+1)/2} \nu^{-(p-1)/2}}$$

thus $J(\nu) \propto \nu^{-a}$, where $\boxed{a = \frac{p-1}{2}}$

Exact expression given by 8.88 & involves many Γ functions

Approx expression if consider avg. Lorentz factor $\langle \gamma \rangle$

$N_e(\gamma) = n_e(\gamma) V$, so $L_{syn} = \frac{4}{3} \sigma_T c U_B V \int_1^\infty \gamma^2 n_e(\gamma) d\gamma$

$$\text{define } \langle \gamma \rangle = \frac{\int_1^\infty \gamma^2 n_e(\gamma) d\gamma}{\int_1^\infty \gamma n_e(\gamma) d\gamma} \to \frac{\langle U_e \rangle}{m_e c^2}$$

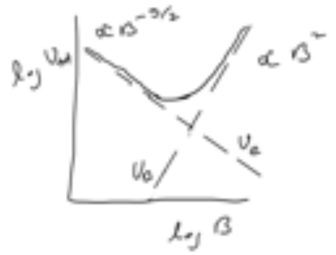
so $\boxed{L_{syn} = \frac{4}{3} \sigma_T U_B U_e V \langle \gamma \rangle \frac{1}{m_e c}}$

product of energy densities in e^- pop. (U_e) & mag. field (U_B)

$\langle \gamma \rangle$ is NOT avg. # dist value of γ, but avg. weighted by E dist.!

If synch. all that's measured, don't

know $U_B + U_e$ indep., just pr...



Minimum Energy (Pressure) Condition

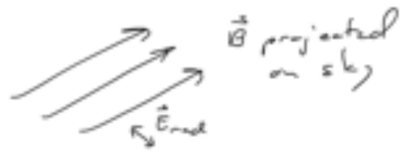
→ Equipartition b/t e^- & B

→ ratio of ions to e^- unknown $\varepsilon_p/\varepsilon_e = 1, 10, 100?$

$U_e \propto B^{-3/2}$ comes from integrating E over some range

- Measure L_ν, not L_{bol}
- Don't know volume
- Don't know filling factor
- Most E in ↓E particles b/c $p \gtrsim 2$

Polarization



$\vec{E}_{rad} \perp \vec{B}_{sky}$

avg. over all α

Fractional polarization $\boxed{\Pi = \dfrac{p+1}{p+7/3}}$

for typical radio spectrum, $a \sim 1$ so $p \sim 3$

& $\Pi \sim 75\%$: highly polarized

Synchrotron self-absorption

For those in rad. proc., might come across the principle of <u>detailed balance</u>

- every emission process has a corresponding absorption process

Think atomic transitions

→ Here, occurs if brightness T_B of radiation approaches "thermal" T of e^-

Although e^- non-thermal, associate a T_e via $\gamma m_e c^2 = 3kT_e$ rel.

[from $P = \mathcal{E}(\gamma_{ad} - 1) \to nkT = \gamma m_e c^2 \left(\frac{4}{3} - 1\right)$]

Brightness T_B comes from BB spectrum:

$$I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT}-1} \approx \frac{2kT_b}{\lambda^2}$$

Rayleigh-Jeans
$\downarrow \nu$

Most emission comes out @ $\nu \simeq \nu_s \propto \gamma^2 \nu_g$
(from $F(x)$, Fig. 8.8)

$$T_e = \frac{\gamma m_e c^2}{3k} = \frac{m_e c^2}{3k}\left(\frac{\nu}{\nu_g}\right)^{\frac{1}{2}}$$

so $T_B \sim T_e$ occurs for a source of size $\Omega \sim \theta^2$:

$$S_\nu = I_\nu \Omega = \frac{2m_e}{3\nu_g^{1/2}} \Omega \nu^{5/2}$$

$$\propto \frac{\theta^2 \nu^{5/2}}{B^{1/2}}$$

$\propto \nu^{5/2}$ is the absorption @ $\downarrow$ freq. when radiation becomes optically thick

☆ Fig. 8.12

In radio, continuum emission is dominated by synchrotron
→ includes the plane + "loops" like the North Polar Spur

$$I(\nu) \propto \nu^{-0.4} \quad (\nu < 400\,\text{MHz})$$
$$\propto \nu^{-0.8\text{-}0.9} \quad (\nu > 400\,\text{MHz})$$

Cosmic Ray e^- spectrum above 10 GeV is $dN = N(E)dE = 700\left(\frac{E}{\text{GeV}}\right)^{-3.3} \frac{e^-}{\text{m}^2\,\text{s}\,\text{sr}}$

dens. is then $dn = n(E)dE$
$$= \frac{4\pi\, dN}{c}$$

$$dn = 2.9\times10^{-5}\left(E/_{\text{GeV}}\right)^{-3.3} dE \quad \frac{e^-}{\text{m}^3}$$

or $\sim 0.3\ e^-/\text{m}^3$ @ 1 GeV

e^- w/ $E = \gamma m_e c^2$ radiate

@ $\nu_c \sim 2.8$ (G) ...

If $B \sim 3\times \mu G$, $E \sim 10\,GeV \rightarrow \gamma \approx \frac{10{,}000}{0.511}$

$\nu\,(10\,GeV) = 2.8 \cdot 100 \cdot 1836^2 \left(\frac{3\times 10^{-6}}{G}\right)$ MHz

$\approx (3.4\times)$ GHz

Estimating the volume emissivity is tricky, but $\sim 3\times10^{-39}$ W$\,m^{-3}$

Use synchrotron emissivity equation along w/ e^- energy density

☆ Fig. 8.14

→ implies $B \sim 6\mu G$, 2-3× ↑ than estimates from rotation measure toward pulsars

↳ RMs are weighted by n_e (could be biased) while synch. is weighted more by ↑B