ASTR 5590 - Chapter 9a

Photoelectric Absorption

$$\sigma_K = \frac{e^{12} m_e^{3/2} Z^5}{192\sqrt{2}\,\pi^5 \epsilon_0^6 \hbar^4 c}\left(\frac{1}{\hbar\omega}\right)^{7/2}$$

if $E = h\nu \gg E_I$ & $\hbar\omega \ll m_e c^2$

→ for ground state electrons (K shell)

Cross-section ∅ below E_I (photon not absorbed) → corresponds to absorption edge

Other levels in atoms have edges as well, if occupied by e^-

$\omega^{-7/2}$ dependence means photons need to be close to E_I to get absorbed

Z^5 means rare, ↑Z elements can dominate absorption, esp. @ ↑E ☆ Fig 9.102

Can define the optical depth (when last scattering occurred) thru a medium as

$$\tau = \int \sigma n_H \, dl$$

→ path length

[illegible] ($^{-3}$)

effective cross-section density cm^{-3}
(cm^2)

$$\tau = 2\times10^{-22}\left(\frac{h\nu}{1\,\mathrm{keV}}\right)^{-8/3} \underbrace{\int n_H\, dl}_{N_H}$$

N_H column depth (cm^{-2})

Typical column density in ISM is 10^{19} (out of plane) to 10^{21} (thru the plane)

@ 1 keV, $\tau \lesssim 1$; @ $E > 1$ keV $\tau \ll 1$

Thomson scattering ($h\nu \ll m_e c^2$ or $\ll m_e c^2$ in center of mom.)

Radiation (unpolarized) incident on a stationary e^- will get scattered

Arguments similar to discussion of accel. of e^- due to passing ion (bremsstrahlung)

Can derive cross-section → effective size of e^- in terms of amount of radiation that gets scattered

$$\sigma_T = \frac{8\pi}{3} r_e^2 = \frac{e^4}{6\pi\epsilon_0^2 m_e^2 c^4}$$

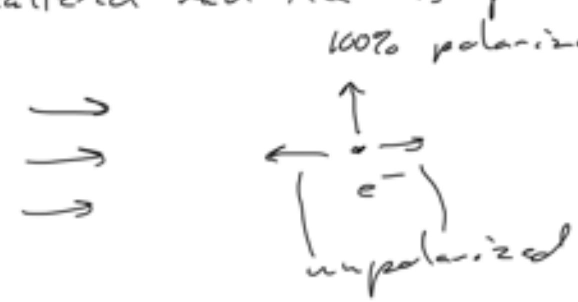
$$= 6.653 \times 10^{-29}\, m^2$$

Has several properties

1) scattering is symmetric : same in back direction as forward

2) scattered radiation (E lost from rad. field) is related to E density of rad. as $-\left(\frac{dE}{dt}\right) = \sigma_T\, c\, u_{rad}$

3) scattered radiation is polarized



4) optical depth of scattering is

$$\tau_T = \int \sigma_T\, n_e\, dx$$

w/ m.f.p. $\lambda_T = (\sigma_T n_e)^{-1}$

Compton scattering — when freq. of photon changes

$$\frac{\omega'}{\omega} = \frac{1 - (v/c)\cos\theta}{1 - \left(\frac{v}{c}\right)\cos\theta' + \left(\frac{\hbar\omega}{\gamma m_e c^2}\right)(1 - \cos\alpha)}$$

θ → angle b/t γ & $\vec{v}_e$

θ': angle after collision

α: angle scattered

↑E photon collides w/ stationary e^-,

have $v=0$, $\gamma=1$

$$\frac{\Delta\lambda}{\lambda} = \frac{\hbar\omega}{m_e c^2}(1-\cos\alpha)$$

Freq. changes are on the order of $\frac{v}{c}$, but E is gained/lost in equal measure by photons on this order

But, photons lose E on the order of $\left(\frac{v}{c}\right)^2$

Crosssection is variable depending on relative Energies, given by Klein-Nishina formula:

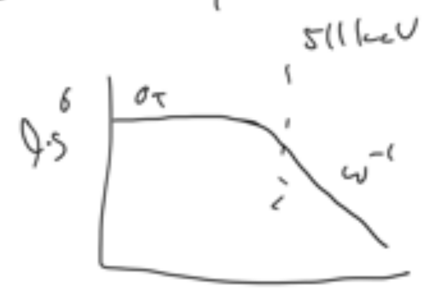
$$\sigma_{K\text{-}N} = \pi r_e^2 \frac{1}{x}\left[\left(1-\frac{2(x+1)}{x^2}\right)\ln(2x+1) + \frac{1}{2} + \frac{4}{x} - \frac{1}{2(2x+1)^2}\right]$$

$$x = \frac{\hbar\omega}{m_e c^2}, \quad r_e = \frac{e^2}{4\pi\epsilon_0 m_e c^2}$$

Can consider $\downarrow$ & $\uparrow$ E limits

$x \ll 1$, $\sigma_{KN} \simeq \frac{8\pi}{3} r_e^2 \approx \sigma_T$

$x \gg 1$, $\propto \frac{1}{\hbar\omega}$



$\hbar\omega \simeq 511$ keV

$x \sim 1$,

γ-rays

$\log E$