ASTR 5590 - Chapter 9b

Inverse Compton Scattering

Fig 9.5 As long as $\gamma h\nu' \ll m_e c^2$,
the Compton interaction in the rest frame of the e^- is Thomson scattering,

so $\boxed{-\left(\frac{dE}{dt}\right)' = \sigma_T c\, u'_{rad}}$

Recall that is invariant: $\left(\frac{dE}{dt}\right) = \left(\frac{dE}{dt}\right)'$

We want $\left(\frac{dE}{dt}\right)$ as a function of u_{rad},
since that's the frame we know its value

Accounting for Doppler shift & arrival time of photons in S',
we find $u'_{rad} = \left[\gamma\left(1 + \frac{v}{c}\cos\theta\right)\right]^2 u_{rad}$

Assuming the radiation field is isotropic,
we just need to integrate over $d\Omega$:

$$du'_{rad} = u_{rad}\,\gamma^2\left[1 + \frac{v}{c}\cos\theta\right]^2 d\Omega$$

$\rightarrow$ Avg, so $\dfrac{\int du'_{rad}}{\int d\Omega}$ $\quad d\Omega = 2\pi\sin\theta\, d\theta$, $\quad \int d\Omega \rightarrow 4\pi$

$$u'_{rad} = u_{rad}\int_0^{\pi}\gamma^2\left[1 + \frac{v}{c}\cos\theta\right]^2 \frac{1}{2}\sin\theta\, d\theta$$

$d\mu = -\sin\theta\, d\theta$

$x = \cos\theta,\ dx \ldots$

$$= \frac{1}{2} u_{rad} \gamma^2 \underbrace{\int_{-1}^{1} \left[1 + \frac{v}{c} x\right]^2 dx}$$

$$x + \frac{v}{c} x^2 + \frac{1}{3}\frac{v^2}{c^2} x^3 \Big|_{-1}^{1}$$

$$2 + 0 + \frac{2}{3}\frac{v^2}{c^2}$$

$$u'_{rad} = u_{rad} \gamma^2 \left(1 + \frac{1}{3}\frac{v^2}{c^2}\right)$$

$1 - \frac{v^2}{c^2} = \frac{1}{\gamma^2}$

$$= u_{rad} \gamma^2 \left(1 + \frac{1}{3}\left[1 - \frac{1}{\gamma^2}\right]\right)$$

$$= u_{rad} \left(\frac{4}{3}\gamma^2 - \frac{1}{3}\right) = \frac{4}{3} u_{rad} \left(\gamma^2 - \frac{1}{4}\right)$$

So, $$\left(\frac{dE}{dt}\right) = \frac{4}{3} \sigma_T c\, u_{rad} \left(\gamma^2 - \frac{1}{4}\right)$$

☆ This is the total energy scattered by the e^- → to get the ↑ in energy, need to subtract off the original energy rate in the radiation field:

$$\sigma_T c\, u_{rad}$$

$$\left(\frac{dE}{dt}\right)_{net} = \frac{4}{3} \sigma_T c\, u_{rad} \left(\gamma^2 - \frac{1}{4}\right) - \sigma_T c\, u_{rad}$$

$$= \frac{4}{3} \sigma_T c\, u_{rad} \underbrace{\left(\gamma^2 - 1\right)}$$

$$\frac{1}{1-\frac{v^2}{c^2}} - \frac{1-\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} = \frac{v^2/c^2}{1-v^2/c^2} = \gamma^2 \frac{v^2}{c^2}$$

$$\boxed{\left(\frac{dE}{dt}\right)_{IC} = \frac{4}{3} \sigma_T c\, u_{rad} \left(\frac{v}{c}\right)^2 \gamma^2}$$

☆ Is this @ all familiar??

$$\left(\frac{dE}{dt}\right)_{syn} = \frac{4}{3}\sigma_T c u_B \left(\frac{v}{c}\right)^2 \gamma^2 \, !$$

☆ Why is the result so similar?

→ E lost by e^- is due to an accel.
in IC, it's an E field (from rad.)
in syn, it's a B field ($\vec{v} \times \vec{B}$)

→ in Thomson scattering, $\left(\frac{dE}{dt}\right)_x \propto |E_x|^2$

& for syn $u_B \propto B^2$

☆ Why γ^2 in above?

→ photon gets Doppler shifted into e^- frame, $\nu' = \gamma \nu$, gets scattered, has same freq. $\nu'' \sim \nu' = \gamma\nu$, then Doppler shift BACK to lab frame, $\nu''' = \gamma \nu'' = \underline{\gamma^2 \nu}$

Doing it properly w/ θ & avg., find

$$\boxed{\nu_{IC} = \frac{4}{3}\gamma^2 \nu_{rad}}$$

If some environment has rel. e^- & some rad. field (starlight in a galaxy, and CMB always), measuring the luminosities of both give the ratio of energy densities

$$\boxed{\frac{L_{IC}}{L_{sync}} = \frac{U_{rad}}{U_B}}$$

Stephan-Boltzmann const

$$U_B = \frac{B^2}{8\pi} \text{ or } \frac{B^2}{2\mu_0}, \quad U_{rad}(BB) = \frac{4\sigma}{c} T_{rad}^4$$