ASTR 5590 - Chapter 9c

<u>Comptonization</u> → when Compton scattering dominates evolution of spectrum

Assume non.-rel., $kT_e \ll m_e c^2$ &

$E = h\nu = \hbar\omega \ll m_e c^2$

→ Thomson scat. valid

$$\frac{\Delta E}{E} = \frac{h\nu}{m_e c^2}(1-\cos\theta)\text{, avg. over }\theta$$

$$\left\langle \frac{\Delta E}{E} \right\rangle_{e^-} = \frac{h\nu}{m_e c^2} \rightarrow E\uparrow \text{ of } e^-$$

$$IC \rightarrow \frac{dE}{dt} = \frac{4}{3}\sigma_T c u_{rad}\left(\frac{v^2}{c^2}\right)\cancel{\gamma^2}^{\,1}$$

(prob. of collision)

cross-section

speed

N_{ph} = # photons scattered per sec → $n_{phot.}\sigma_T c$

density

$$n_{phot.} = \frac{u_{rad}}{h\nu}$$

$$\frac{dE}{dt} = \frac{4}{3}N_{ph}h\nu\left(\frac{v^2}{c^2}\right)$$

So, ΔE gained per photon is, on average

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{4}{3}\frac{v^2}{c^2}$$

If e^- are in thermal eq., $\frac{1}{2}m_e\langle v^2\rangle = \frac{3}{2}kT_e$

so $\left\langle \frac{\Delta E}{E} \right\rangle_{ph} = \frac{4kT_e}{m_e c^2}$ → $E \uparrow$ of photon

Total $\left\langle \frac{\Delta E}{E} \right\rangle_{ph,net} = \left\langle \frac{\Delta E}{E} \right\rangle_{ph} - \left\langle \frac{\Delta E}{E} \right\rangle_{e^-}$

$$= \frac{4kT_e - h\nu}{m_e c^2}$$

If $h\nu > 4kT_e$ → E transferred to e^-
$h\nu < 4kT_e$ → E transferred to γ

Basically, more energetic comp. gives some E to less energetic one

Consider case where e^- hotter, $4kT_e \gg h\nu$

$$\left\langle \frac{\Delta E}{E} \right\rangle_{per\ collision} = \frac{4kT_e}{m_e c^2}$$

- a region w/size l + #dens. n_e has an optical depth $\tau_e = n_e \sigma_T l$

If $\tau_e \gg 1$, random walk for photons escaping

$$l \sim N^{1/2} \lambda_e \qquad (\lambda_e = [n_e \sigma_T]^{-1})$$

mfp

$$N = \left(\frac{l}{\lambda_e}\right)^2 = \left(\frac{\tau_e}{n_e \sigma_T} n_e \sigma_T\right)^2 = \tau_e^2$$

If $\tau_e \ll 1$, basically only get 1 scat. or less, w/ $N = \tau_e$

For a spectrum to be distorted by Comptonization, $4y \gtrsim 1$ where

$$y = \frac{kT_e}{m_e c^2} \max\left(\underset{<1}{\tau_e}, \underset{>1}{\tau_e^2}\right)$$

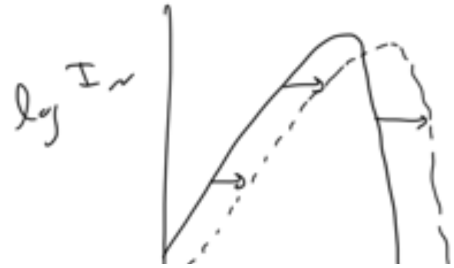
y parameter : Compton optical depth

If $y \gg 1$, photons achieve BB spectrum

If $y \ll 1$, then $y = \dfrac{kT_e}{m_e c^2} \underbrace{n_e \sigma_T l}_{\tau}$

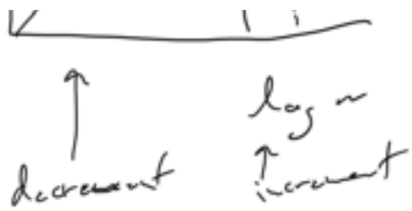
– consider e^- hotter than radiation (cluster gas & CMB photons)

N_{ph} conserved by geometry, but E transferred to each



Produce a distortion of the spectrum

Ex: 9.11



$$\frac{\Delta I_\nu}{I_\nu} = -2y$$

Clusters of galaxies, w/ $T_e \sim 10^8$ K,

$y \lesssim 10^{-5}$, so change in CMB small

If integrate over entire cluster, defining

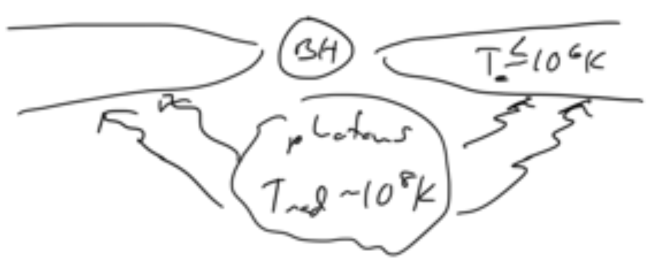
$$y = \frac{kT_e}{m_e c^2}\sigma_T \int n_e \, dl$$

$$\rightarrow \quad Y = \int y \, dA = \frac{kT_e}{m_e c^2}\sigma_T \int n_e \, dl \cdot dA$$

$p = n_e kT$

$$= \frac{\sigma_T}{m_e c^2}\int n_e kT_e \, dV \cong \frac{\sigma_T}{m_e c^2} \overbrace{PV}^{\text{thermal } E \text{ of cluster gas}}$$

What if photons have ↑E than e^-?



- Give E to e^-
- Create a "Compton Hump"