ASTR 5590 - Dispersion measure & FR measure

Recall the plasma freq. is the natural freq. of a plasma, where separated charge restores itself

$$\nu_p = \left(\frac{e^2 n_e}{4\pi^2 \epsilon_0 m_e}\right)^{1/2} \simeq 9 \left(\frac{n_e}{1\,cm^{-3}}\right)^{1/2} \text{kHz}$$

Radio freq. generally > 10 MHz, so

$$\nu_p/\nu \ll 1$$

Group velocity of light thru the medium is

$$v_{gr} = c\left(1 - \left(\nu_p/\nu\right)^2\right)^{1/2}$$

$$\approx c\left[1 - \frac{1}{2}\left(\frac{\nu_p}{\nu}\right)^2\right]$$

If you have a pulse of emission, the pulse travels to us w/a speed that depends on freq.

Arrival time:

$$t_a = \int_0^l \frac{dl}{v_{gr}} = \int_0^l \frac{dl}{c}\left[1 + \frac{1}{2}\left(\frac{\nu_p}{\nu}\right)^2\right]$$

$$= \frac{l}{c} + \frac{e^2}{8\pi^2 \epsilon_0 m_e c} \frac{1}{\nu^2} \int_0^l n_e \, dl$$

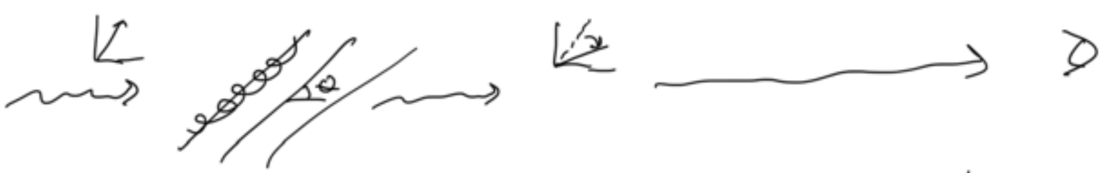
$$\simeq 4.2\times10^{9}\left(\frac{\nu}{\text{Hz}}\right)^{-2}\left[\int\left(\frac{n_e}{m^{-3}}\right)\frac{dl}{\text{parsecs}}\right]\text{s}$$

Allows measurement of $\int n_e\,dl$ → possible toward ~2000 Galactic pulsars

If n_e assumed, can solve for distance

FR Measure

Because the plasma freq. + gyro freq. are both much less than radio observation freq., linear polarized emission gets rotated along the direction of the magnetic field



Decompose $\vec{E}$ field vector in left/right handed ell. polarizations, which rotate in opposing senses thru the plasma

$$n^2 = 1 - \frac{(\nu_p/\nu)^2}{1 \pm (\nu_g/\nu)\cos\theta}$$

n: index of refraction

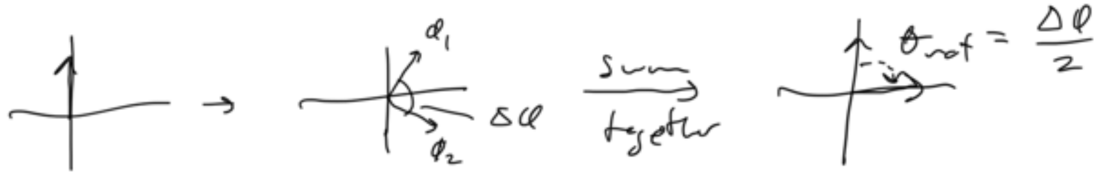
For $\nu_p/\nu \ll 1$ + $\nu_g/\nu \ll 1$, the diff.

$\nu_p^2\nu_g \quad \theta$

in n is $\Delta n = \frac{\cdots}{\nu^3} \cos\theta$

Phase diff. $\Delta\varphi$ b/t modes is

$$\Delta\varphi = \frac{2\pi\nu}{c}\Delta n \, dl$$



$$\Delta\theta = \frac{\pi \nu_p^2 \nu_s}{c\nu^2} \cos\theta \, dl$$

$$\nu_s = 2.8 \times 10^{10} \left(\frac{B_{\parallel}/\cos\theta}{T}\right) \text{Hz}, \quad \nu_p = 9\left(\frac{n_e}{m^{-3}}\right)^{1/2} \text{Hz}$$

$$\theta = \frac{\pi}{c\nu^2}\int_0^l \nu_p^2 \nu_s \cos\theta \, dl$$

$$= 8.1 \times 10^3 \left(\frac{\lambda}{m}\right)^2 \int_0^l \left(\frac{n_e}{1\,m^{-3}}\right)\left(\frac{B_{\parallel}}{T}\right)\frac{dl}{pc}$$

☆ Product of $n_e B_{\parallel}$ along l.o.s.

Can get avg. B along l.o.s. if measure both rot. measure & disp. measure

$$\langle B_{\parallel} \rangle \propto \frac{RM}{Disp.\,M} \propto \frac{\int n_e B_{\parallel} \, dl}{\int n_e \, dl}$$

BUT, $\nearrow$ is weighted by the density along the l.o.s., so the $\uparrow n_e$ regions contribute more to $\langle B_{\parallel} \rangle$

→ if n_e & $B_\parallel$ correlate, then

$\langle B_\parallel \rangle$ will be biased

& won't reflect true value of B