ASTR 5590 - Pulsars II

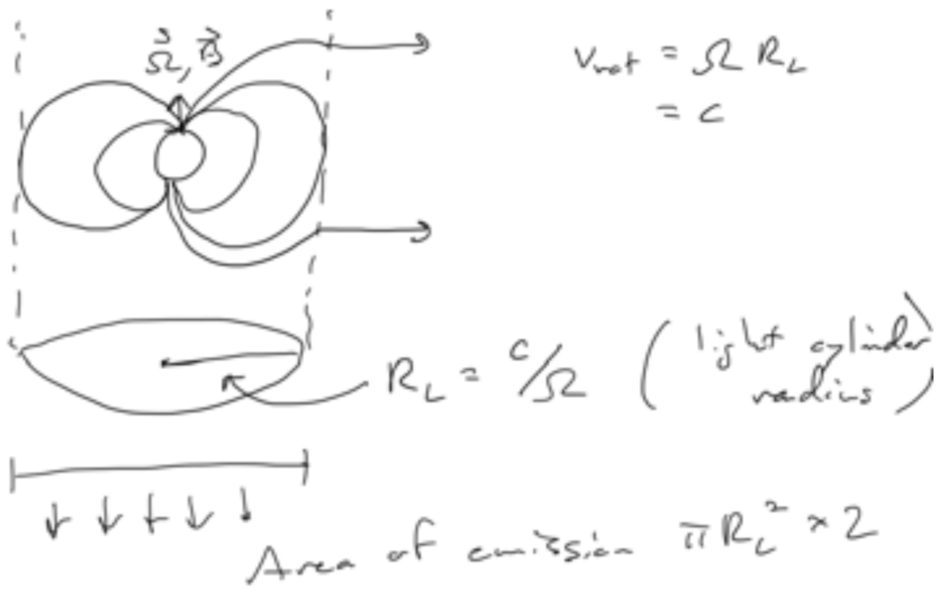
Show Crab Nebula expansion

As mentioned, not powered by SN
(luminosity too high for age)
but is powered by $\left(\frac{dE}{dt}\right)_{rot}$

So... what causes ↗ ?

From dipole, have $\frac{dE}{dt} \approx \frac{2\mu^2 \Omega^4}{3c^3}$

- simplify, consider Ω & B aligned



$V_{rot} = \Omega R_L$
$= c$

$B \sim \frac{\mu}{r^3}$, $B(r = R_L) \approx \frac{\mu}{R_L^3}$

Energy density is then $\varepsilon_B = \frac{B^2}{8\pi}$

$= \frac{\mu^2}{8\pi R_L^6}$

So total E emitted is

$$\frac{dE}{dt} \sim \mathcal{E}_B \cdot A \cdot v$$

↳ rate of oscillation where emission occurring $= c$

$$\sim \frac{\mu^2}{8\pi R_L^6} 2\pi R_L^2 c = \frac{\mu^2 c}{4 R_L^4}$$

$$@ R_L,\ \Omega = \frac{c}{R_L},\ \frac{dE}{dt} \sim \frac{\mu^2 \Omega^4}{c^3}$$

Get same form as misaligned

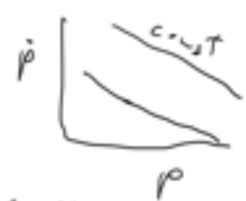
↳ E loss same regardless of α

In general, misaligned like 13.15

Get a <u>Pulsar Wind Nebula</u>

- carries away most rot. E
- $\vec{\Omega}$ aligned w/ motion (torque)
- $P\dot{P} = \text{func}(I, \mu, \alpha)$

so $P\dot{P} = \text{const}$



Can define a "braking index" n

$$\boxed{\dot{\Omega} \propto -\Omega^n}$$: how quickly is period slowing down given its period

$$\dot\Omega = -K\Omega^n$$

$$\ddot\Omega = -nK\Omega^{n-1}\cdot\dot\Omega = -nK\Omega^n\frac{\dot\Omega}{\Omega} = n\frac{\dot\Omega^2}{\Omega}$$

$$n = \frac{\ddot\Omega\,\Omega}{\dot\Omega^2} \rightarrow$$ need to measure 2nd deriv.

$$\Omega = \frac{2\pi}{P},\quad \dot\Omega = -\frac{2\pi\dot P}{P^2},$$

$$\ddot\Omega = -2\pi\left(\frac{\ddot P}{P^2} - 2\frac{\dot P^2}{P^3}\right) = -\frac{2\pi}{P^2}\left(\ddot P - 2\frac{\dot P^2}{P}\right)$$

$$n = \left(2\frac{\dot P^2}{P} - \ddot P\right)\frac{1}{\cancel{P^2}}\,\frac{P^4}{\dot P^2} = 2 - \frac{\ddot P P}{\dot P^2}$$

$= 3$ for mag. dipole

measured $n = \begin{cases} 2.515 \pm 0.005 & \text{Crab} \\ 2.837 \pm 0.001 & \text{B1509-58} \\ 1.81 \pm 0.07 & \text{B0540-69} \\ 3.0 \pm 0.1 & \text{J1119-6127} \end{cases}$

HERE

Assume n is a const., can derive its age: $\dot\Omega = -K\Omega^n = \frac{d\Omega}{dt}$

$$K\int_0^\tau dt = -\int_{\Omega_0}^{\Omega}\Omega^{-n}\,d\Omega$$

$$K\tau = \frac{1}{n-1}\left[\Omega^{1-n} - \Omega_0^{1-n}\right]$$

current   ang. vel. born with

If $n>1$ & $\Omega_0 \gg \Omega$, then

$$\tau = \frac{\Omega^{1-n}}{K(n-1)} = \frac{\Omega}{\dot\Omega(1-n)}$$

$$\tau = \frac{1}{n-1}\frac{P}{\dot{P}}$$

$$\dot{\Omega} = \frac{d}{dt}\left(\frac{2\pi}{P}\right)$$
$$= -2\pi\frac{\dot{P}}{P^2} = -\Omega\frac{\dot{P}}{P}$$

Setting $n=3$, get the spindown age or characteristic age of pulsar

Crab, $\tau \sim 1400\,yr$, close to true age ($\sim 1000\,yr$)

→ assumes K is a const.

For most radio pulsars, $\tau \gtrsim 10^6\,yr$

Can plot pulsars on a $P-\dot{P}$ diagram

→ pulsars of a given age will follow the line $\dot{P} = \frac{1}{2\tau}P$

→ if we take $B = B_{pole} = \frac{2\mu}{R^3}$

Then $B \propto (P\dot{P})^{1/2}$, so

$$\dot{P} = B^2 P^{-1}$$

$$\log\dot{P} = -\log P + 2\log B$$

$P-\dot{P}$ diagram

- born w/ ~10^{12} G field
- moves along B track until
  P large enough rot. E loss
  makes undetectable since

$$-\left(\frac{dE}{dt}\right)_{rot} = -I\Omega\dot{\Omega} = -4\pi I\dot{P}P^{-3}$$