ASTR 5590 - Pulsars III

Pulsar Magnetosphere

Complex problem to find a solution for

- take $\alpha = 0$ (Ω & B aligned)
- B field is frozen in

- expect an electric field to therefore be present: Why?

Get an induced electric field from motion: $\vec{E}_i = \vec{v} \times \vec{B}$

But, charges move to remove field, so

$$\vec{E} + (\vec{v} \times \vec{B}) = \vec{E} + \left[(\vec{\Omega} \times \vec{r}) \times \vec{B} \right] = 0$$

↖ charges moving to cancel out $\vec{E}_i$

☆ Recall, what is conductivity of plasma?
→ high; for NS assume ∞

For a dipole, this leads to an external electrostatic potential (quadrupolar)

$$\Phi_E(r,\theta) = -\frac{B_0 \Omega R^5}{6 r^3} \left(3\cos^2\theta - 1\right)$$

(mks)

$B = \frac{\mu_0 \mu}{\ldots}$ (mks) or $\frac{2\mu}{\ldots}$ ( )

$\quad 4\pi R^3 \quad \ldots \quad R^3$ (cgs)

↳ polar field, $\sim 10^{12}$ G

$$E = \nabla \varphi, \quad \text{so } \nabla \varphi \sim \frac{B_0 \Omega R^5}{r^4}$$

$$E(R) \sim B_0 \Omega R$$

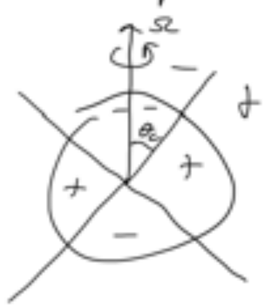
Crab pulsar surface E&M vs. grav.

$$\frac{e(\vec{v} \times \vec{B})}{GM m/R^2} \sim \frac{e B_0 \Omega R}{GM m/R^2} \approx 10^{12}$$

so charge motion determined only by E → e^- will be dragged off the surface → origin of PWN

HERE?

B/c quadrupolar, get charges separated by a critical angle



$$3\cos^2\theta_c - 1 = 0$$

$$\theta_c \sim 55°$$

→ charge separates to create E that cancels out induced field E_i

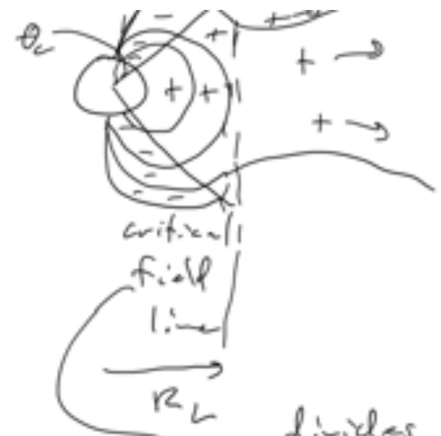
Remember the light cylinder, where you get open field lines

B lines inside LC trap particles

– open field lines allow particles to escape (causing E loss), but...

★ Fig. 13.23

$r = r_0 \sin^2\theta$

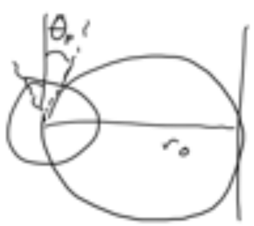


↖ radius at line @ equator

$r_0 > R_L \rightarrow$ open field line

$R_L = c/\Omega$

R_L divides whether + or − charge leaves pulsar



angle associated w/ last closed field line gives size of conical beam

$$R = R_L \sin^2\theta_p$$

− assume small, $\theta_p \approx \left(\frac{R}{R_L}\right)^{1/2} = \left(\frac{\Omega R}{c}\right)^{1/2}$

$$\theta_p = 1^\circ \left(\frac{R}{10\,\mathrm{km}}\right)^{1/2} \left(\frac{P}{1\,\mathrm{s}}\right)^{-1/2}$$

so for $P \sim 0.1$ s, $\theta_p \sim 3^\circ$

Nicely explains short pulse width in radio, assuming radio emission produced at surface

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BUT, what is the radio emission??

− can't be incoherent given the brightness temperature of emission (Rayleigh Jeans tail of BB)

$$T_B = \frac{\lambda^2}{2k}\left(\frac{S_\nu}{\Omega}\right)$$

S_ν ← flux density

Ω ← solid angle of pulse

$T_B > 10^{23}$ K ... must be coherent

$T_B \sim 10$ K → too energetic,
particle E above esc. vel.

Need coherent emission, like a laser,
which allows bunches of particles
to emit simultaneously

e.g., some kind of "maser" process
in the plasma w/in the magnetosphere

$T_B \sim 10^{18}$ K in this case, comparable
to radio transmitters on Earth!

@ ↑E, like X-rays, the brightness
temp. is lower, $\sim 10^{11}$ K (10 MeV),
so incoherent emission is allowed

To get e^- to v. ↑E, need to accel.
them (need to get rid of + charges)
→ inner/outer gaps potential sites

Fig. 13.15

e^- more easily ripped off, radiate
synchrotron until $\vec{v} \parallel \vec{B}$

- emits via curvature radiation
(cyclotron, but due to curve of B)

- these photons interact w/ strong B
+ produce e^-, e^+ : pair cascade

$$\gamma + B \rightarrow e^- e^+ \rightarrow \gamma\gamma \rightarrow e^- e^+ \rightarrow \cdots$$

e^- + e^+ are accel. in opposite directions

So don't annihilate right away

↳ cascades could produce coherent emission needed in radio

called a "spark", creates the gap region above the pole

↳ sparks could cause fine structure seen in pulses on $t \lesssim 100 \mu s$

Also could explain why pop. has

$B \sim 10^{12} G$, $P \sim 1 s$

↳ need this for pair cascades to occur

→ predicts the death line:

$\quad \gtrsim 10^{11} \left(P \right)^{-2}$