Homework 1

Due date: Jan 28, 2019 (Mon, in class). No late homework.

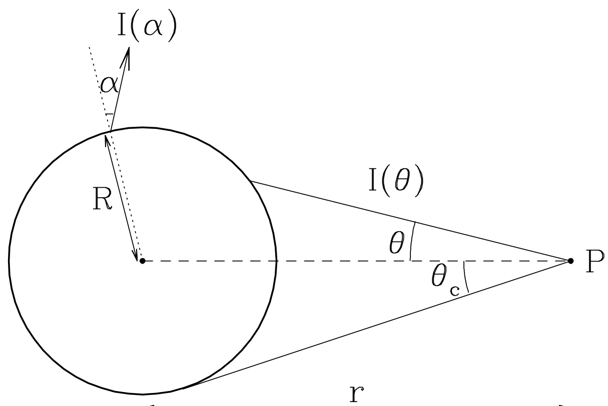
1. (20 pts) Intensity per Frequency Interval vs Intensity per Wavelength Interval

(a) If I_ν is the intensity per unit frequency interval and I_λ is the intensity per unit wavelength interval, show that $\nu I_\nu = \lambda I_\lambda$.

(b) Apply the above result to the blackbody radiation – use $B_\nu(T)$ we derived in class to derive $B_\lambda(T)$.

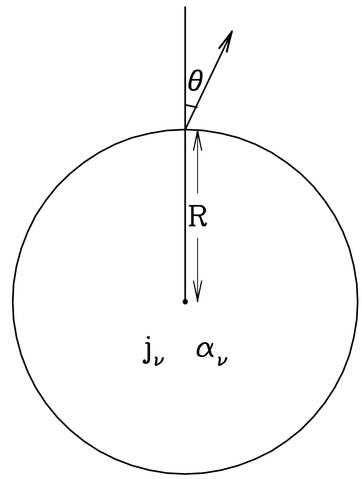
2. (20 pts) Inverse Square Law for an Isotropic Source

In Chapter 1.3 of Rybicki&Lightman, it is proved that the flux from a uniformly bright sphere (i.e., $I = constant$) follows the inverse square law. In fact, in the more general case of an isotropic source even with $I \neq constant$, the flux also follows the inverse square law. A proof based on the energy conservation is given in Chapter 1.2 of Rybicki&Lightman. Now prove it by starting from the intensity integration. Similar to that in Chapter 1.3, $F = \int I \cos\theta d\Omega = \int_0^{2\pi} d\phi \int_0^{\theta_c} I(\theta)\sin\theta\cos\theta d\theta$ with $\theta_c = \sin^{-1}(R/r)$, and in this case $I(\theta)$ is not necessarily a constant. Finish the proof. (**Hint:** at any point of the surface of the sphere, I is only a function of α, $I = I(\alpha)$, where α is the angle with respect to the radial direction. See the plot below.)



3. (60 pts) Radiative Transfer in a Uniform Sphere and Photon Escape Probability

Consider a sphere of radius R with uniform and isotropic emission coefficient j_ν and absorption coefficient α_ν. Define the optical depth τ_R across the radius as the optical radius of the system. There is no external radiation field, i.e., all the radiation in this problem comes from the sphere.



(a) Express τ_R in terms of R and the relevant coefficient.

(b) At the surface of the sphere, what is the specific intensity $I_\nu(R, \theta)$ at an angle θ with respect to the radial direction?

(c) At any point of the surface, what is the flux F_ν?

(d) What is the luminosity L_ν that comes out of the sphere?

(e) What is the net momentum flux p_ν at any point of the surface?

(f) If there is no absorption, what would be the flux $F_{\nu,0}$ at the surface?

(g) The photon escape probability P_{esc} is defined as the ratio $F_\nu/F_{\nu,0}$. Write down the expression of P_{esc} in terms of τ_R and show that it is of the order of $1/\tau_R$ for $\tau_R \gg 1$.

(h) At radius $r < R$, with the corresponding optical depth across r denoted as τ, what is the specific intensity $I_\nu(r, \theta)$ at an angle θ with respect to the radial direction?