Homework 2

Due date: Feb 11, 2019 (Mon, at class time). No late homework.

1. (40 pts) Stokes Parameters

Two waves propagating along the z-axis are superposed. One wave is linearly polarized along the x-axis and the other is right-hand circularly polarized. The intensity of the linearly polarized wave is half of that of the circularly polarized one.

(a) If the two waves are monochromatic with the same frequency and there is a phase difference ϕ between the electric field of the linearly polarized wave and the x-component of the electric field of the circularly polarized one, find the Stokes parameters for the superposed wave. What is the degree of polarization of the superposed wave?
(b) If the two waves are instead quasi-monochromatic and independent, find the Stokes parameters for the superposed wave and decompose them into a completely polarized and a completely unpolarized component. What is the degree of polarization of the superposed wave?

2. (50 pts) Retarded Potential of a Uniformly Moving Charge

A particle of charge q is moving on the z-axis towards $+\hat{z}$ direction with a constant velocity u. Suppose that the particle passes $z = 0$ at $t = 0$. Consider a field point $\mathbf{r}$ with distance $r = |\mathbf{r}|$ to the origin and angle θ between $\mathbf{r}$ and the $+\hat{z}$ direction. In the following, we will consider the scalar potential $\phi(\mathbf{r}, t) = \phi(r, \theta, t)$ at $t = 0$.

(a) If the field point is in front of the particle, i.e., $\theta = 0$, what is the corresponding retarded time $t_{\rm ret}$ for $t = 0$? What is the retarded position $\mathbf{r}'$ of the particle? What is the distance R from the field point to the particle's retarded position? What is the $\kappa = 1 - \mathbf{n} \cdot \boldsymbol{\beta}$ factor in the retarded potential? What is the retarded potential $\phi(r, \theta = 0, t = 0)$?
(b) Repeat the calculations in (a) for $\theta = \pi$, i.e., the field point is behind the particle.
(c) Repeat the calculations in (a) for $\theta = \pi/2$, i.e., the field point is beside the particle.
(d) More generally, compute the retarded time $t_{\rm ret}$ for the field point (r, θ) at $t = 0$. Then compute the retarded potential $\phi(r, \theta, t = 0)$. Does there exist a forward-backward symmetry such that $\phi(r, \pi - \theta, t = 0) = \phi(r, \theta, t = 0)$?

Note that your results should be expressed in terms of q, r, θ, and $\beta = u/c$.

3. (10 pts) An electron is moving along a trajectory $\mathbf{r}_0(t)$. At a field point $\mathbf{r}$ and time t, is it possible to have two values of retarded time $t_{\rm ret}$ correponding to two different positions on the electron's trajectory? If your answer is "yes", please give a concrete example, i.e., the form of $\mathbf{r}_0(t)$ and the situation for the two values of $t_{\rm ret}$. If your answer is "no", please give your reasoning and proof.