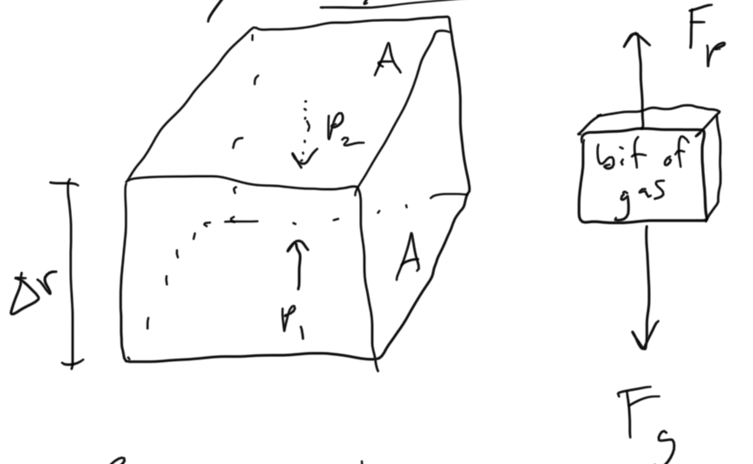
ASTR 3070 Week 10 (Ch.14 & 15)

Stellar Atmospheres & Classification

Atmospheres in general (that of the Earth, Sun, etc.) have a structure determined by **hydrostatic equilibrium**



$$F_p = F_g$$

Pressure is a force per area

To keep the gas stationary, the difference in pressure across the box must equal F_g

$$\underbrace{(P_2 - P_1)}\, A = -\frac{G M(<r)\, \overbrace{\rho A \Delta r}^{m}}{r^2}$$

If box small enough

$$\Delta P$$

$$dP = -\frac{GM\rho}{r^2}\,dr$$

$$\underbrace{\frac{dP}{dr}}_{\text{pressure gradient}} = -\underbrace{\frac{GM(<r)\rho}{r^2}}_{\text{gravity}}$$

☆ For a spherical dist. w/ density that only depends on radius, all mass interior can be thought of as in the center, and the grav. force of all mass exterior cancel out, hence $M(<r)$

Luminosity Classes

- w/o distance hard to get class, but turns out <u>line widths</u> depend on class
- narrower lines in I (supergiant) than V (dwarf / MS): why?

Surface gravity, g, is ↓ b/c $R\uparrow$,

& thus the pressure is lower, so the
lines have less <u>pressure broadening</u>

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} = -g\rho$$

Line comes from a shell dr that
corresponds to the optical depth
$\tau = 0 \to 1$, $\tau = n\sigma x$, or here,
can write as $d\tau = -n\sigma\, dr$ (as $\tau\uparrow$, $r\downarrow$)

Can rewrite the cross-section as a quanti
per mass (in this shell), called the
<u>opacity</u>: $\kappa = \frac{\sigma}{\text{mass}}$, or $\rho\kappa = n\sigma$

$$d\tau = -\rho\kappa\, dr$$

$$\frac{dP}{dr} = \frac{dP}{-d\tau/\rho\kappa} = -g\rho$$

$$\frac{dP}{d\tau} = \frac{g}{\kappa} \to P \sim \frac{g}{\kappa}\tau$$

Since $\tau \sim 1$, $P \sim \frac{g}{\kappa}$ & κ is rough
constant in stars, so $\downarrow g$, $\downarrow P$, narrow
lines

☆ Also, gas in stars generally follows the ideal gas law

$$P = nkT = \frac{\rho k T}{\mu m_p}$$

Density is $\frac{M}{V}$, but particles have diff. individual masses, so use 3 categories

$$\rho = \rho_H + \rho_{He} + \rho_{metal}$$

→ all elements w/ $Z > 2$ are called "metals", even though O, N, Ne, etc. are not technically metals

Express the ratio of each cat. by the total density, define

$$X \equiv \rho_H / \rho$$

$$Y \equiv \rho_{He} / \rho$$

$$Z \equiv \rho_{metal} / \rho = 1 - X - Y$$

$t \sim \rho_{metal}/\rho \sim \ldots$

For the Sun, we observe

$$X_\odot = 0.734, \quad Y_\odot = 0.250, \quad Z_\odot = 0.016$$

This is very similar to the primordial abundance, $X \sim 0.75 / Y \sim 0.25 / Z = 0$

Often we care about # density

$$n = \frac{\rho}{\mu m_p}$$

$$\mu_H = \begin{cases} 1 & \text{neutral} \\ \frac{1}{2} & \text{ionized} \end{cases}, \quad \mu_{He} = \begin{cases} 4 & \text{netral} \\ 4/3 & \text{ionized} \end{cases}$$

$$\mu(\text{ionized}) = \frac{\rho}{n m_p}$$

$$= \left(2X + \frac{3}{4}Y + \frac{1}{2}Z\right)^{-1}$$

$$\mu_\odot(\text{ion.}) = 0.60$$

Stellar Interiors

Just like atmospheres, the interior structure is governed by HSE

$$\frac{dP}{dr} = -\frac{G M(<r) \rho(r)}{r^2}$$

By twisting calculus to our ends, can rewrite as

$$\frac{\Delta P}{\Delta r} \approx -\frac{G \langle M \rangle \langle \rho \rangle}{\langle r \rangle^2}$$

$$\langle M \rangle \sim \frac{M_\odot}{2}$$

$$\langle \rho \rangle \sim \frac{M_\odot}{\frac{4}{3}\pi R_\odot^3} = \rho_\odot = 1400 \frac{k}{m}$$

$$\sim 1.4 \times \rho_{H_2O}$$

$$\langle r \rangle \sim \frac{R_\odot}{2}$$

$$\Delta P = P_{surf} - P_c = 0 - P_c$$

$$\Delta r = R_\odot - 0$$

$$P_c \approx \frac{8\pi}{3} G \rho_\odot^2 R_\odot^2$$

What about T_c? Can use

ideal gas law!

$$P(r) = \frac{\rho(r) k T(r)}{\mu m_p}$$

$$T_c = \frac{2 G M_\odot \mu_\odot m_p}{R_\odot k}$$

$\mu_\odot \simeq 0.60$ $\boxed{T_c \approx 3 \times 10^7 K}$

(Correct analysis gives $2\times \downarrow T_c$)

For stars on the main sequence
w/ $M < 1.66 M_\odot$

$$R/R_\odot = 1.06 (M/M_\odot)^{0.945}$$

$$\approx M/M_\odot$$

Since $T_c \propto \frac{M}{R} \mu$, + μ same for any star (since H dominates mass)

$$T_c \propto 0.6 \frac{M}{R} \propto 0.6 \frac{\cancel{M}}{\cancel{M}} \frac{M_\odot}{R_\odot}$$

$\propto$ const

This const. temperature determines the main sequence — what sets T_c?

Energy Generation

- in the 1800s, no nuclear physics (atomic structure unknown)
- gravity understood (Newtonian), so Helmholtz & Kelvin considered the formation of the Sun from a contracting gas cloud — its light would then be the conversion of grav. pot. E to heat

For a spherical object,

$$U = -q \frac{GM^2}{R} \quad \left(q \text{ depends on } \rho(r) \right)$$

This is the E that must be removed to collapse an object from $r = \infty$ to $r = R$ that has mass M

If the Sun's light comes from this energy, & we assume it has been emitting it at a constant rate...

$$U \rightarrow J \qquad L \rightarrow J\,s^{-1}$$

Can estimate the total lifetime of the Sun!

$$t_{KH} = \frac{U_\odot}{L_\odot} \approx \underline{50 \text{ Myr}}$$

Geologists thought the Earth was older, but Kelvin used this estimate to cow them, setting back geology as a field for <u>decades</u> — almost 100 yr!

<u>Moral</u>: Physicists who don't respect the science of other fields are evil

↳ e.g., physicists making epidemiological models for COVID-19 spread at same university

In the 1930s, nuclear fusion

source

recognized as energy) ...

$$4H \rightarrow 1\,He^4$$

$$\left.\begin{array}{l} 4m_p = 6.6905\times10^{-27}\,kg \\ 1m_{He} = 6.6447\times10^{-27}\,kg \end{array}\right\} 0.0458\times10^{-27}\,kg \text{ difference}$$

Where does mass go?

$$E = mc^2!$$

$$E = \Delta m c^2 = 4.1\times10^{-12}\,J$$

Small amount of E, but Sun has lots of protons! Assuming it's entirely protons

$$N_H = \frac{M_\odot}{m_p} \approx \frac{2\times10^{30}\,kg}{1.67\times10^{-27}\,kg} \approx 10^{57}$$

If fuse all into He ($4H \rightarrow 1He$), then have Δm per reaction, & there are $\frac{N_H}{4}$ reactions, releasing

$$E_{fus} = \frac{N_H}{4}\Delta E = \frac{10^{57}}{4}\,4\times10^{-12}\,J = \underline{10^{45}\,J}$$

This is ~2000x more E than provided

by $U_\odot$, so $t_{fus} = \frac{E_{fus}}{L_\odot} \approx 100$ Gyr

Stars like the Sun only convert ~10% of their H to He, so actual lifetime is 10^{10} yr = 10 Gyr

Since $\tau \propto \frac{M}{L}$ & $L \propto M^4$,

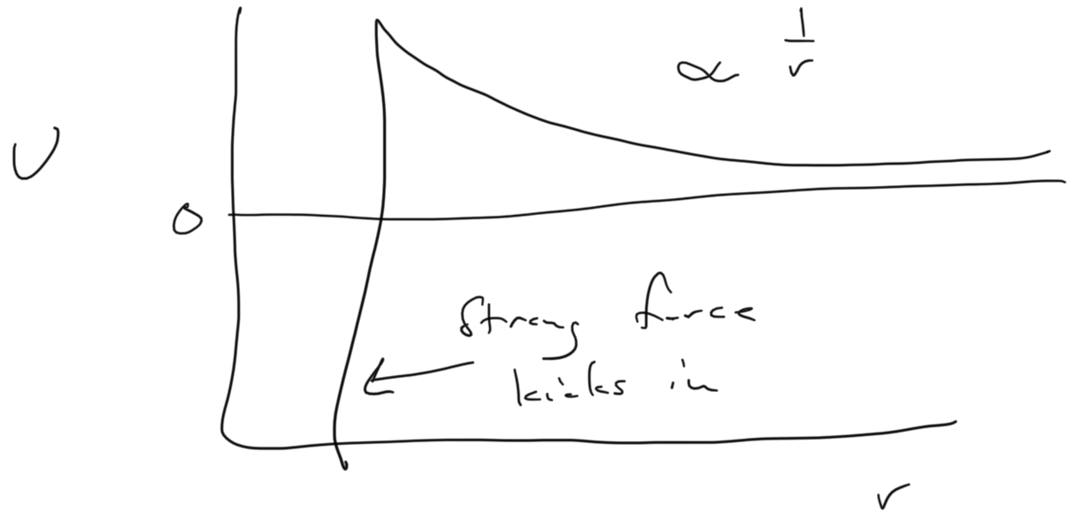
$$\tau \approx 10\,\text{Gyr} \left(\frac{M}{1 M_\odot}\right)^{-3}$$

for main sequence stars

Fusion reactions

How can 2 protons fuse? The repulsive Coulomb force is $\propto \frac{1}{r^2}$ — if protons were point-like, the force would repeal them always

But p^+ aren't — they're made of quarks governed by the strong force which is VERY attractive



Need enough energy to get close enough for strong force to take over

$r \sim 10^{-15}\,m$

$$U \approx \frac{e^2}{4\pi\epsilon_0 r} \approx 1.4\ MeV$$

Do protons in the center of the Sun have this much kinetic E?

$$\langle E \rangle = \frac{3}{2} k T_c \approx 2\ keV$$

Nope! So how does fusion occur?

Particles are not just particles, but also waves (QM saves the day!)

$$\lambda = \frac{h}{p}\ ,\ p = m_p v$$

$v \sim 0.2\,\%c$

$$\lambda_{prot} \approx 10^{-13}\,m$$
$$\sim 100\, r_{SF}$$

Essentially, the proton's position is uncertain, & has a chance of getting close enough even though classically that's impossible

quantum tunneling

So, how exactly do these reactions create He in the core?

$\boxed{\text{PP chain}}$ $T_c < 1.8\times10^7\,K$ (eg, Sun)

$$p + p \rightarrow {}^2H + e^+ + \nu_e$$
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$p \rightarrow n$, positron carries away extra charge, neutrino to conserve e^- quantum #

energy from mass diff.

$${}^2H + p \rightarrow {}^3He + \gamma$$

[illegible]

$$^3H + p \rightarrow {}^3He + \gamma$$

$$^3He + {}^3He \rightarrow \boxed{^4He} + p + p$$

$$4p \rightarrow {}^4He + 2\gamma + 2e^+ + 2\nu_e$$

$$e^+ + e^- \rightarrow 2\gamma$$ (antimatter annihilation)

ν_e only interact via the weak force, which has a very $\downarrow$ cross-section, so Sun is essentially transparent to them & they escape

The γ-rays produced in the core take b/t 10,000-100,000 years to escape (b/c they scatter a lot — have a small m.f.p.). They are in LTE as they travel, so lose E as they travel, becoming visible [illegible] at the surface.

photons on

H → He releases converts 0.7% of the mass to energy

He → bigger elements releases more!

For more massive stars, where $T_c > 1.8 \times 10^7$ K, H → He via the CNO cycle (C, N, O atoms used in intermediary steps, but no net change in their #s – they're catalysts)

CNO cycle gets 1 additional γ-ray

$$^{12}C + \boxed{p} \rightarrow {}^{13}N + \gamma \rightarrow {}^{13}C + e^+ + \nu_e + \boxed{p} \rightarrow {}^{14}N + \gamma + \boxed{p} \rightarrow$$

$$^{15}O + \gamma \rightarrow {}^{15}N + e^+ + \nu_e + \boxed{p} \rightarrow {}^{12}C + {}^{4}He$$

To fuse He → C, need the triple alpha process

$T_c > 10^8 K$ as well as dense environment

$$^4He + {}^4He \to {}^8Be + \gamma$$

↳ decays quickly,

but if it hits another ^4He first

$$^4He + {}^8Be \to {}^{12}C + \gamma$$

$$3\,He \to 1\,C$$