# Model Universes

ASTR/PHYS 4080: Intro to Cosmology
Week 4
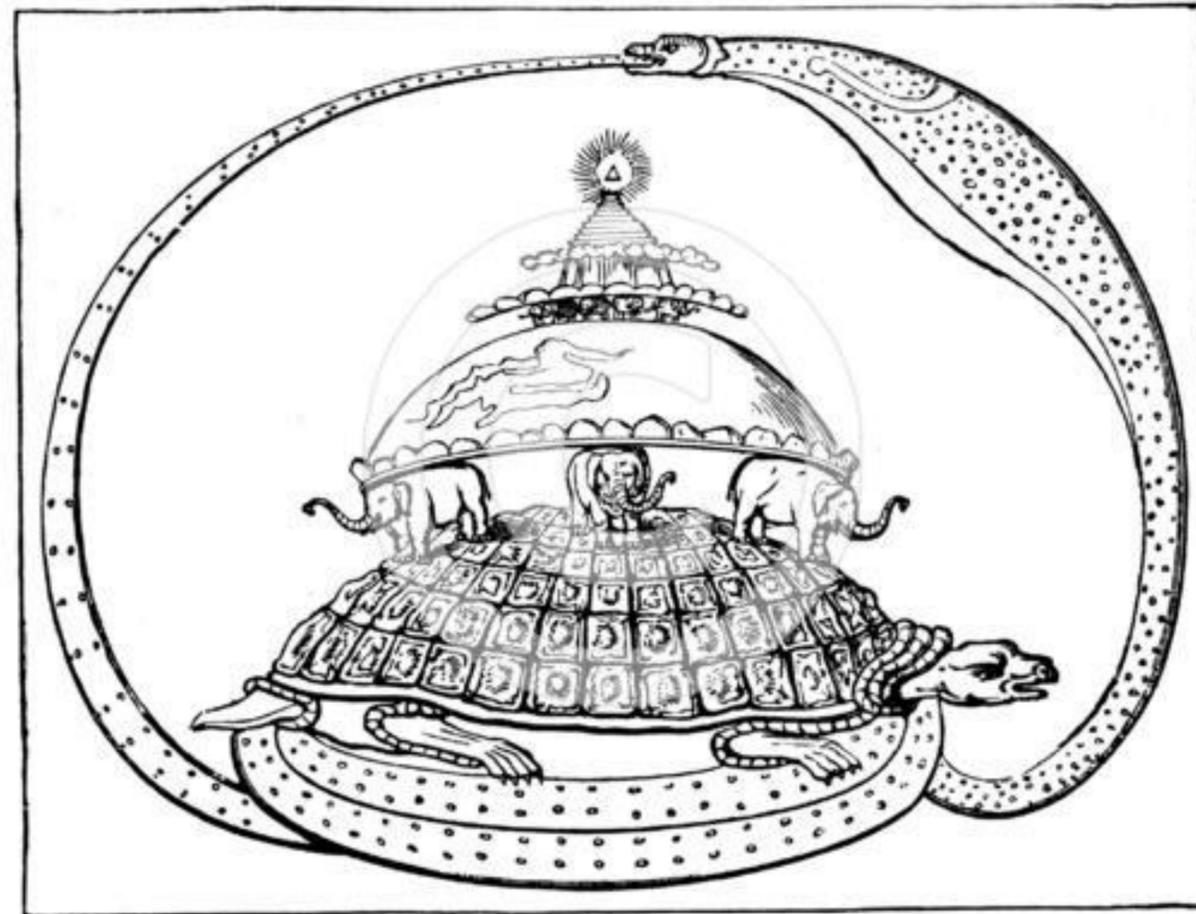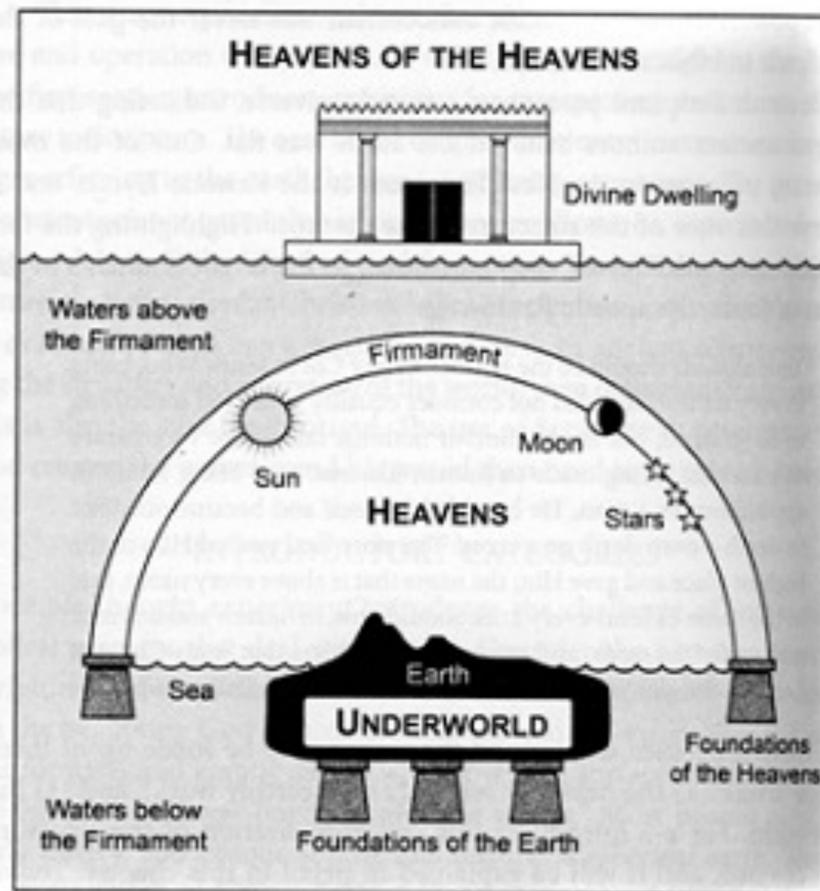
From https://commonphysics.wordpress.com/tag/physics/, where the tag line is "In science there is only physics, all the rest is stamp collecting." Such hubris is why we're still stuck with string theory.

Evolution of Components

$$\varepsilon = \sum_i \varepsilon_i$$

$$P = \sum_i w_i \varepsilon_i$$

Matter
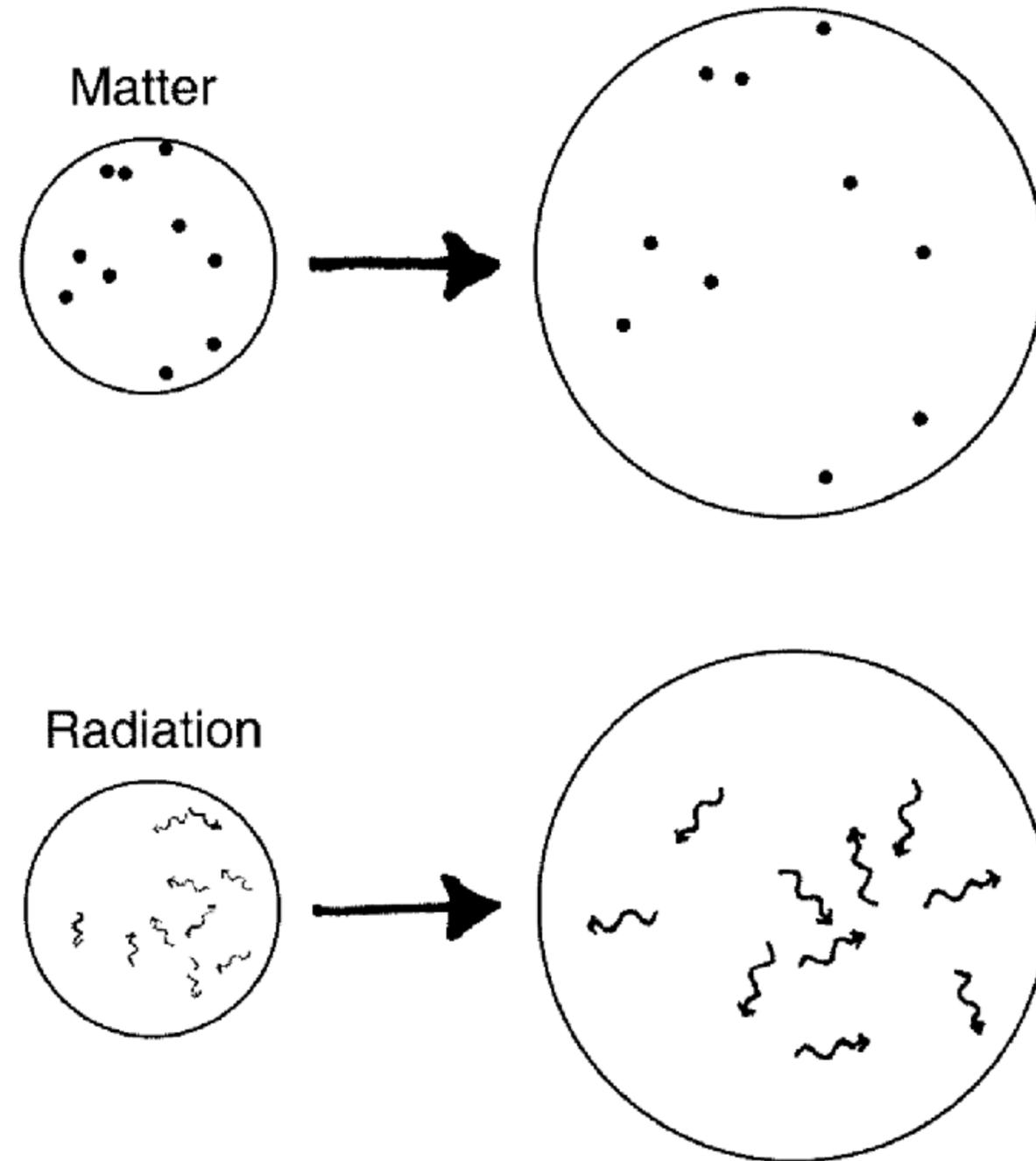
Radiation

Relativistic Component:
"Radiation" = Photons + Neutrinos
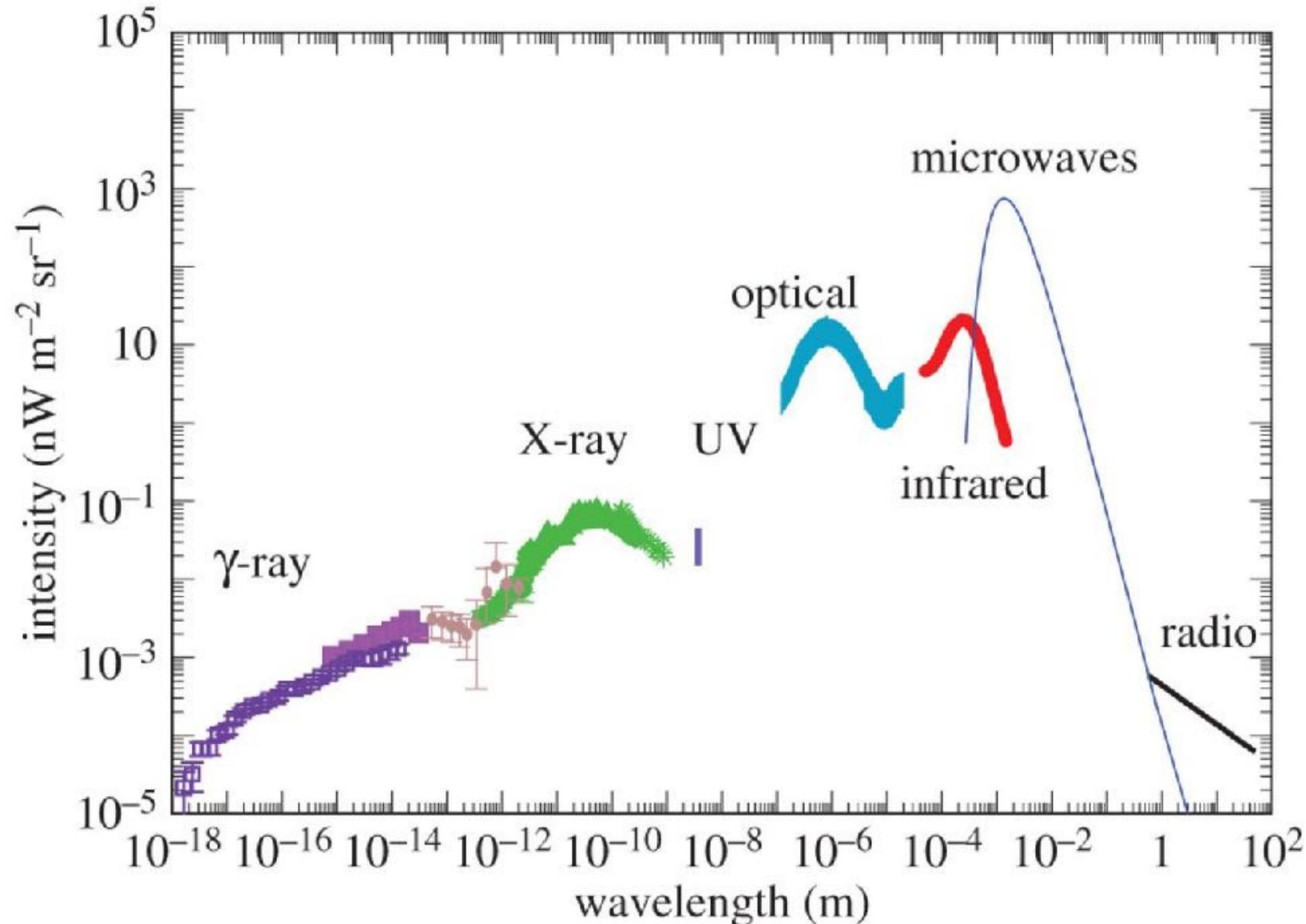
CMB dominates energy density of photons in universe today

Don't know the mass of neutrinos, but energy density dominated by momentum and decoupling in early universe. Calculated to be:

$$\varepsilon_\nu = \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \varepsilon_{\mathrm{CMB}} = 0.227 \varepsilon_{\mathrm{CMB}}$$

True for each neutrino flavor, so total is 3x above.

$$\varepsilon_{\mathrm{CMB},0} = \alpha T_0^4 = 0.2606 \ \mathrm{MeV} \ \mathrm{m}^{-3}$$

Relativistic, Non-rel., & Λ Components

Radiation

$$\varepsilon_{\mathrm{CMB},0} = \alpha T_0^4 = 0.2606 \text{ MeV m}^{-3}$$

$$\varepsilon_{\mathrm{crit},0} = 4870 \text{ MeV m}^{-3}$$

$$\Omega_{\mathrm{r},0} = \frac{1.681\ \varepsilon_{\mathrm{CMB},0}}{\varepsilon_{\mathrm{crit},0}} = 0.9 \times 10^{-5}$$

Matter: $\qquad \Omega_{\mathrm{m},0} \approx 0.31$

Dark Energy: $\qquad \Omega_{\Lambda,0} \approx 0.69$

Using evolution of each component with a, can compute when matter-dark energy and matter-radiation components were comparable

Model Universes

Can now solve for a(t) generically, if not necessarily analytically:

$$\dot{a}^2 = \frac{8\pi G}{3c^2} \sum_i \varepsilon_{i,0} a^{-1-3w_i} - \frac{\kappa c^2}{R_0^2}$$

- Empty
- Matter only
  - classic case: open, closed, flat
- Radiation only (+curvature)
- Lambda only (+curvature)
- Various Combinations!

Empty

Only 1 Constituent in a Flat Spacetime
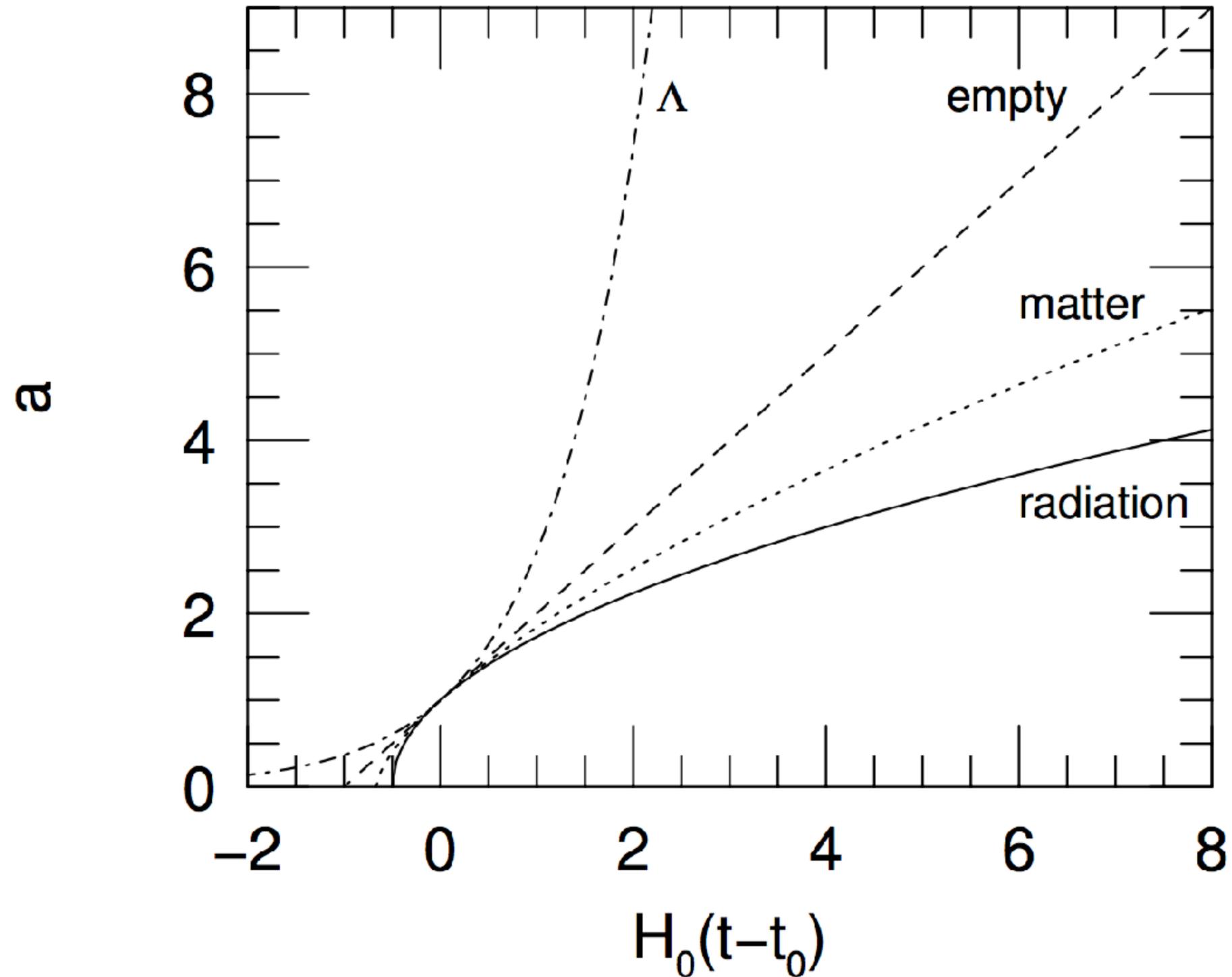
Only 1 Constituent in a Flat Spacetime
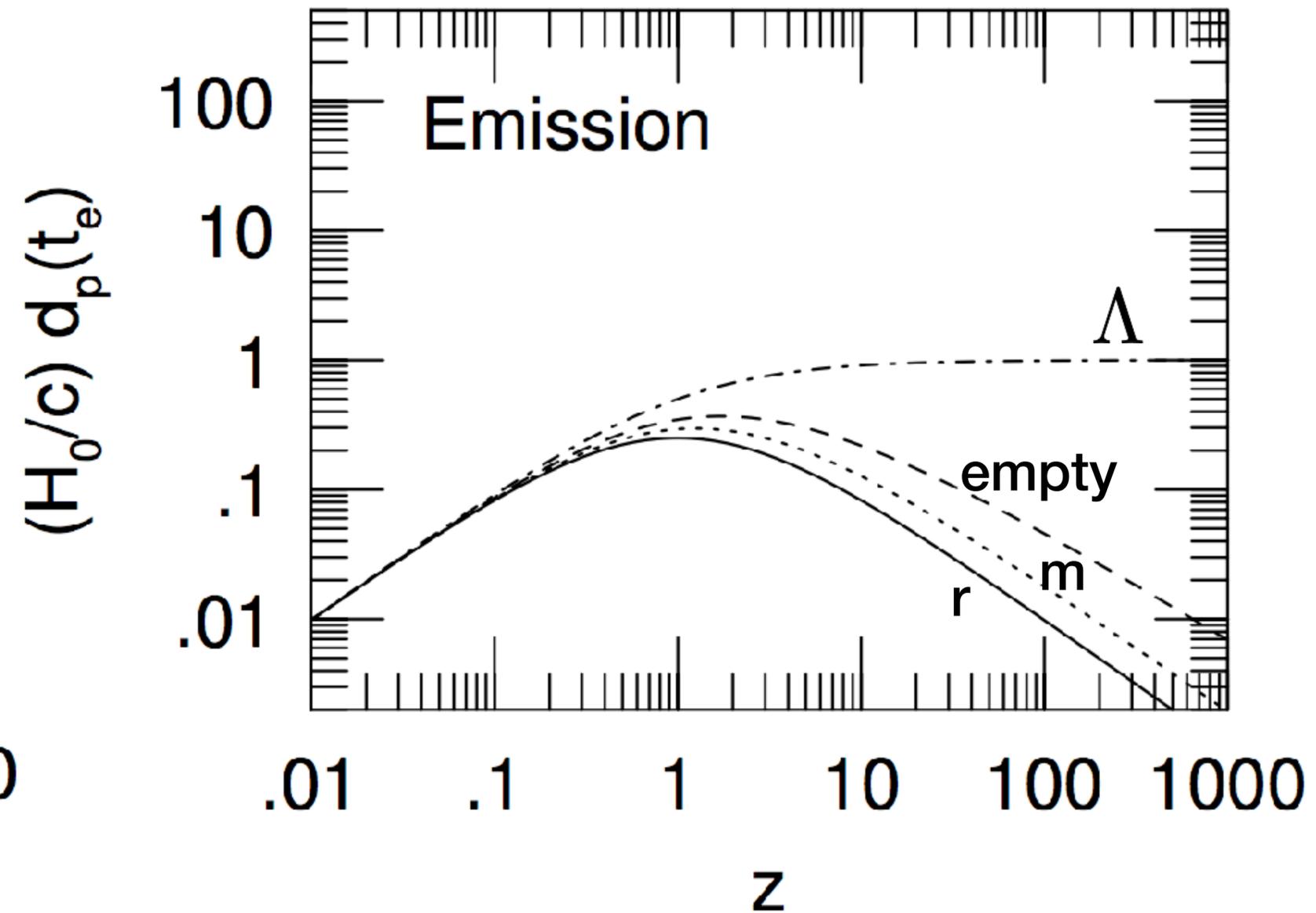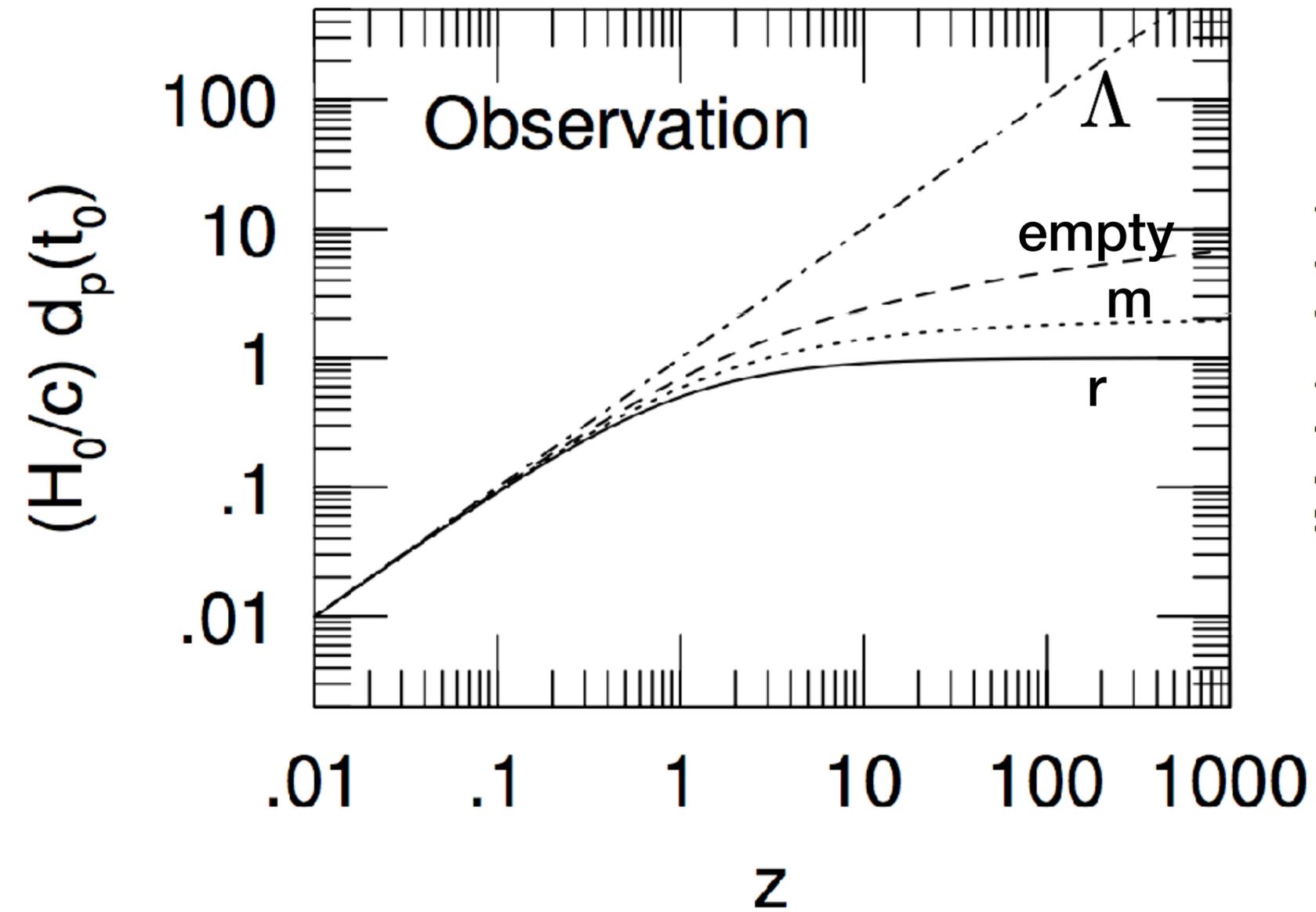
Classical Cosmology: Matter + Curvature
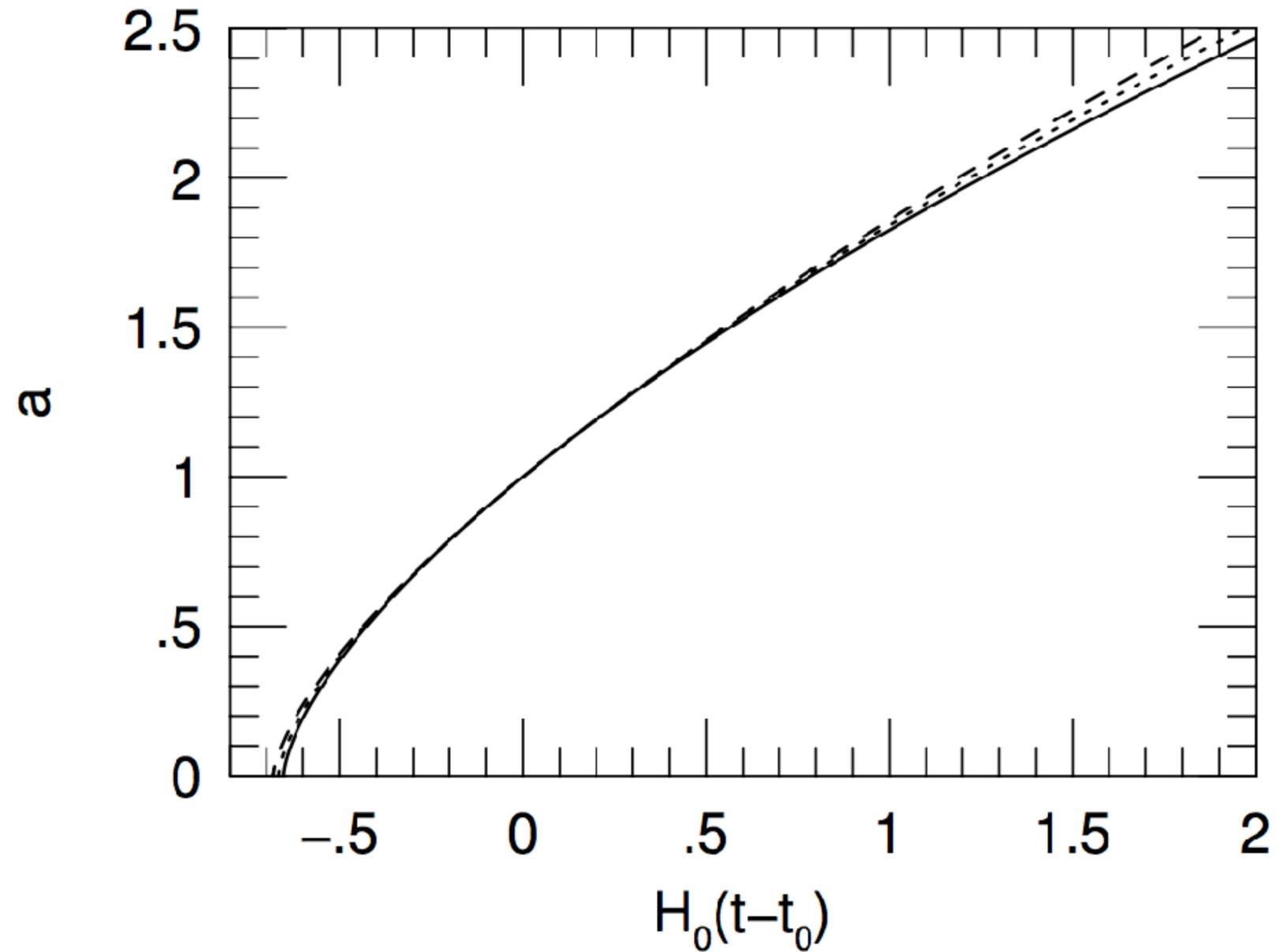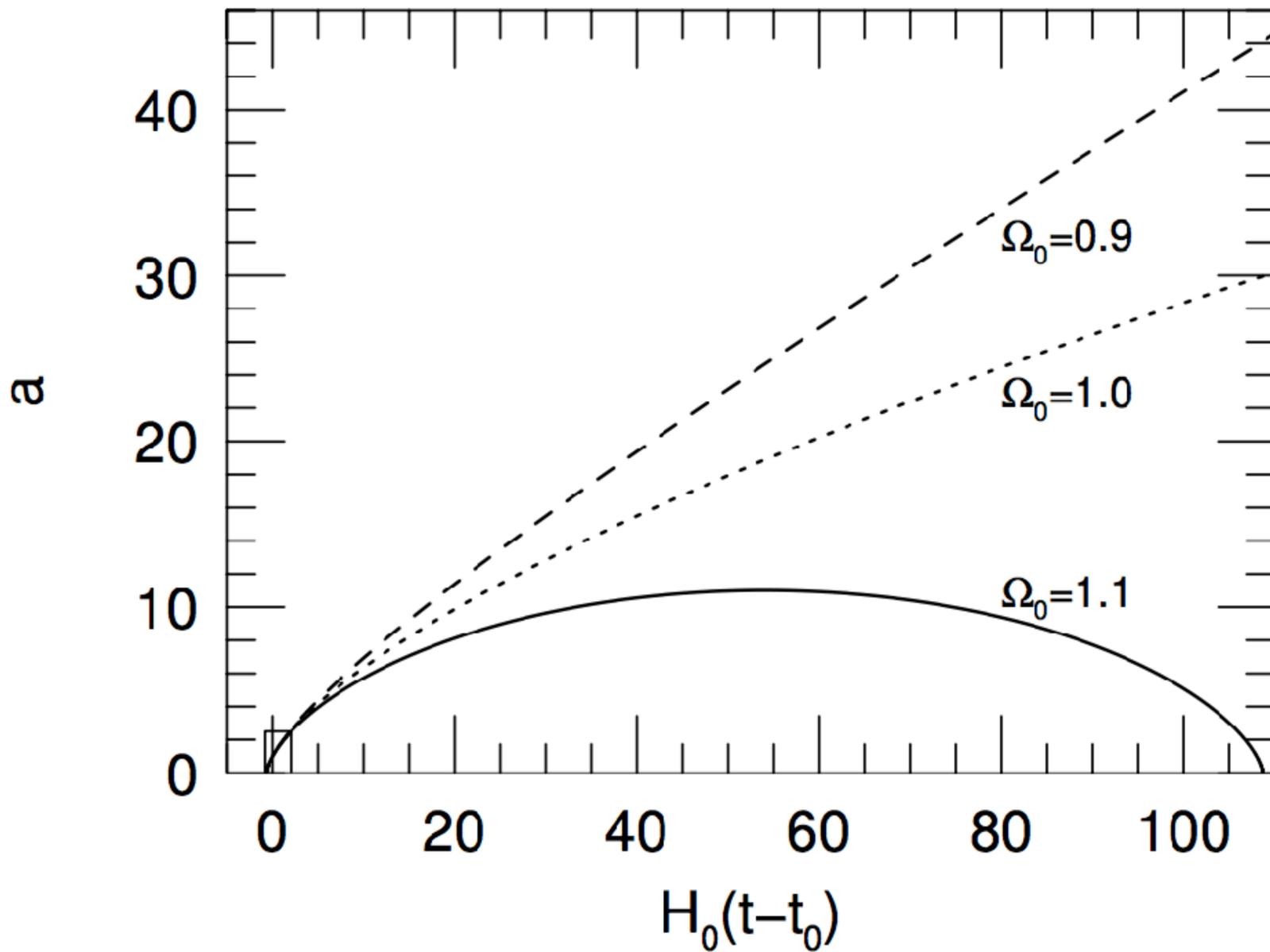
Classical Cosmology: Matter + Curvature
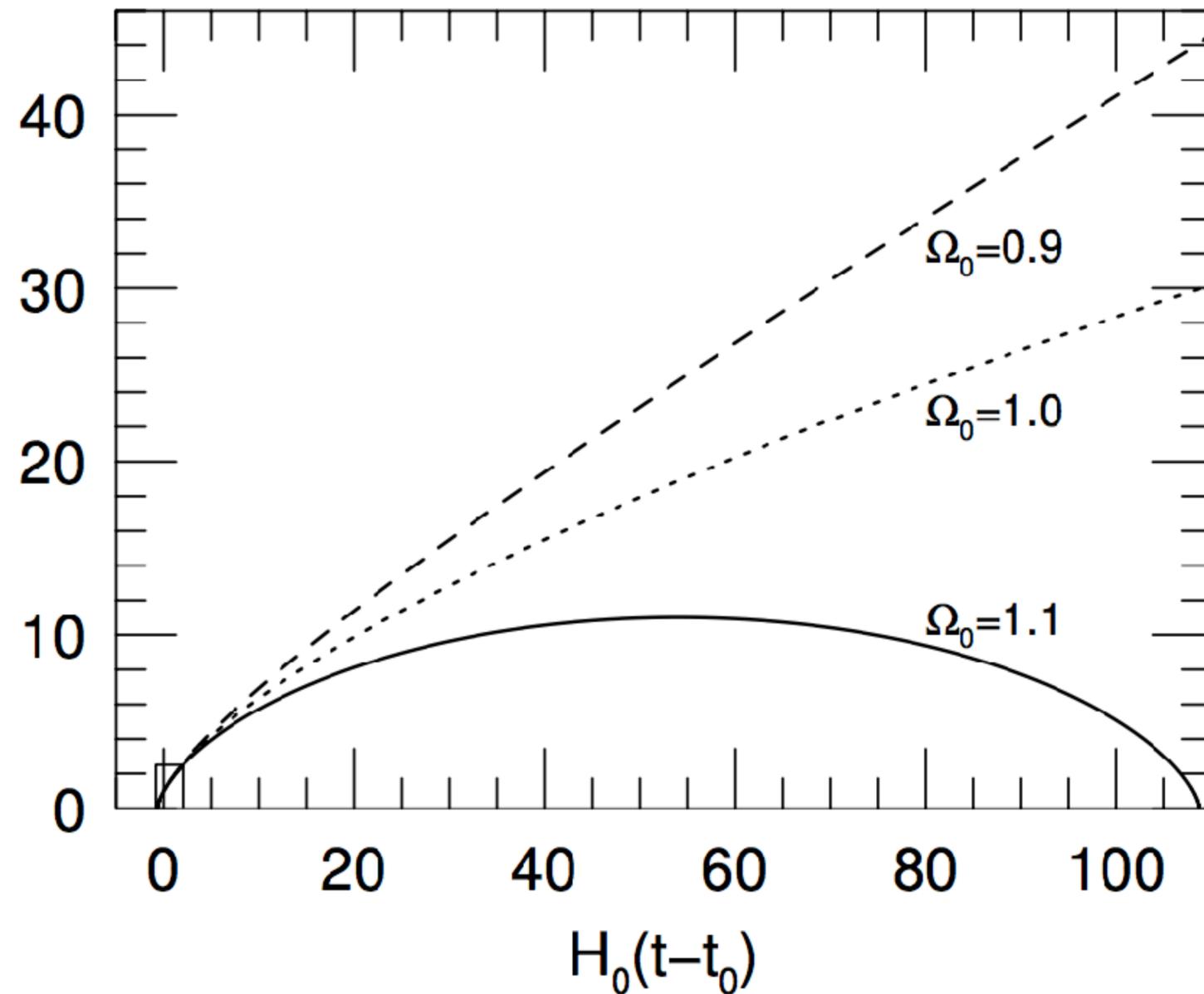
$$\kappa = 0, \ \Omega_0 = 1; \ a(t) = (t/t_0)^{2/3}$$

$$\kappa = +1, \ \Omega_0 > 1; \ a(\theta) = \frac{1}{2}\frac{\Omega_0}{\Omega_0 - 1}(1 - \cos\theta)$$

$$t(\theta) = \frac{1}{2H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}}(\theta - \sin\theta)$$

$$\kappa = -1, \ \Omega_0 < 1; \ a(\eta) = \frac{1}{2}\frac{\Omega_0}{1 - \Omega_0}(\cosh\eta - 1)$$

$$t(\eta) = \frac{1}{2H_0}\frac{\Omega_0}{(1 - \Omega_0)^{3/2}}(\sinh\eta - \eta)$$

Graph axes: a versus $H_0(t - t_0)$; curves labeled $\Omega_0 = 0.9$, $\Omega_0 = 1.0$, $\Omega_0 = 1.1$

Density —> Destiny

Flat: Matter + Cosmological Constant
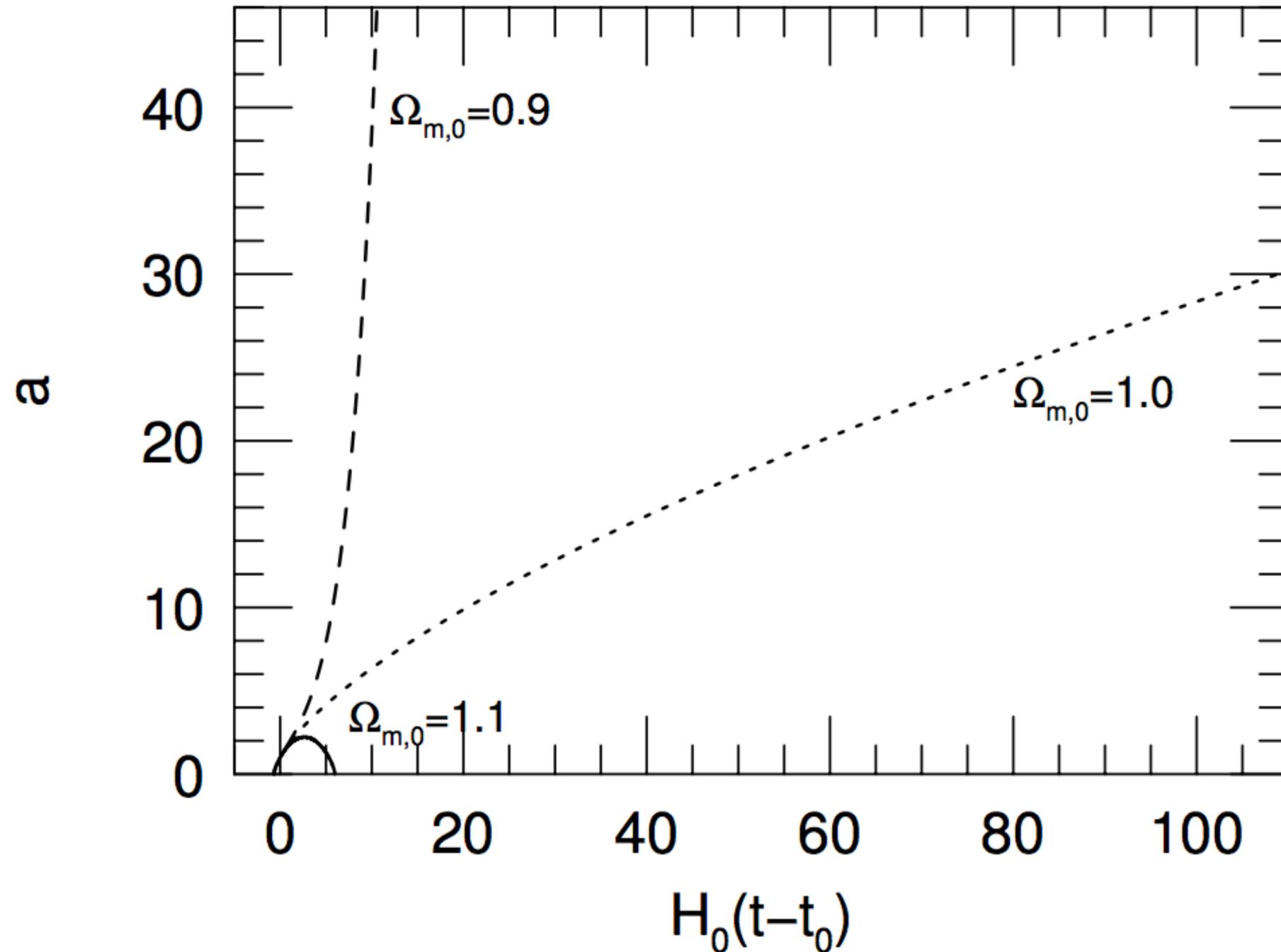
Matter + Lambda + Curvature

Benchmark Model

Benchmark Model