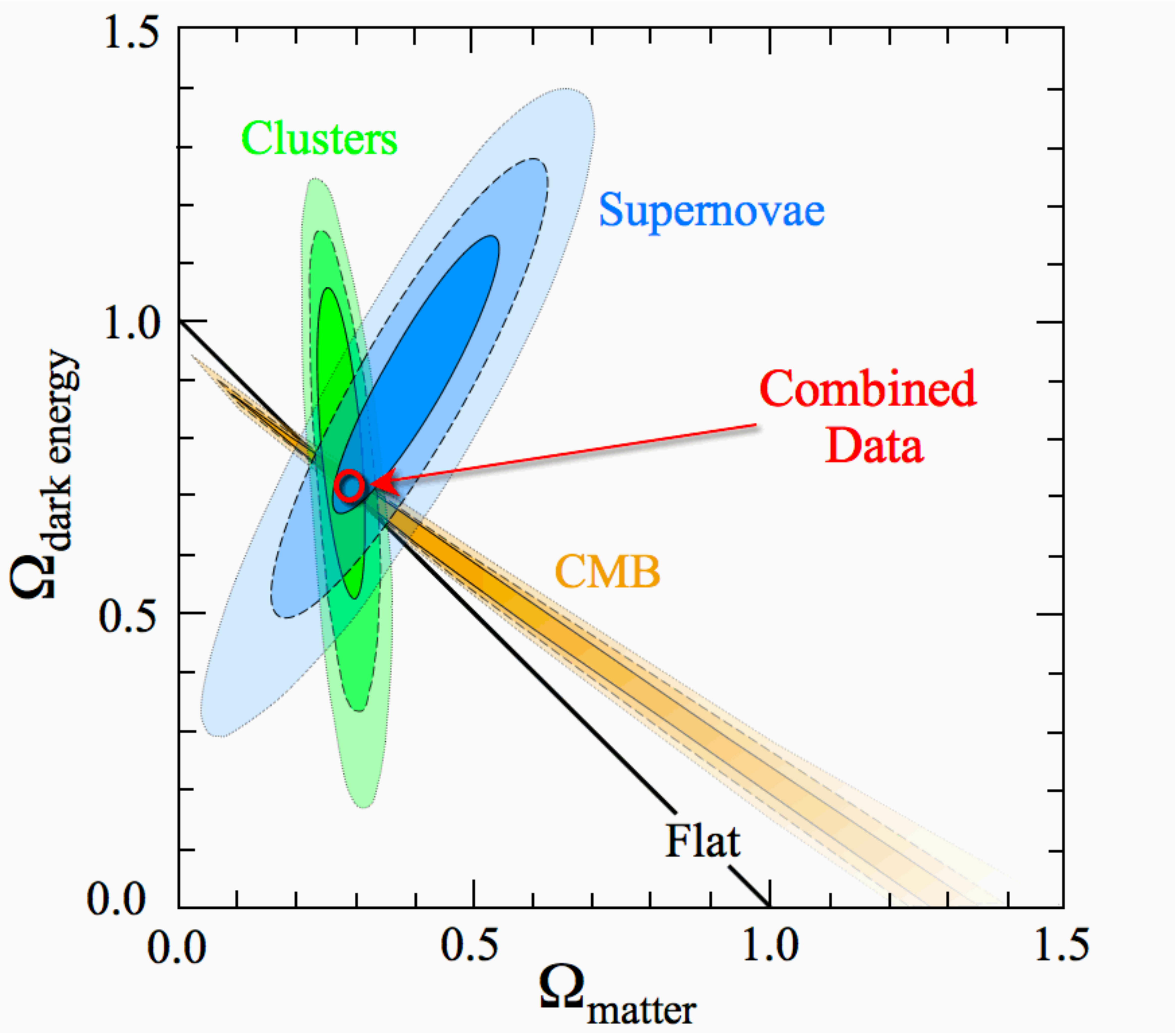
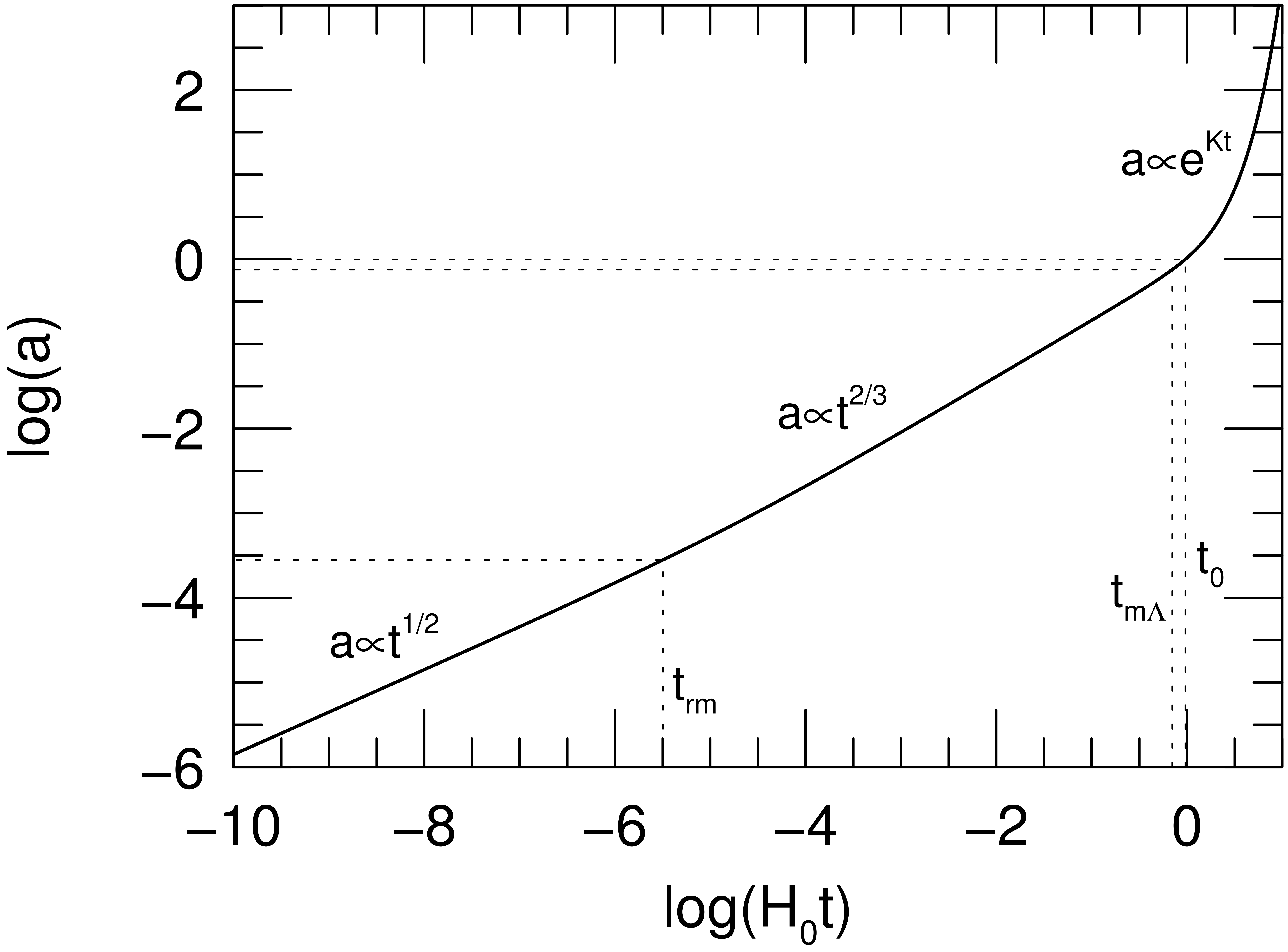
Grand Summary

The Concordance: 1998-2018



Theory

Benchmark Model



$$\kappa = 0$$

$$\Omega_r + \Omega_m + \Omega_\Lambda = 1$$

$$\Omega_r = \Omega_\gamma + \Omega_\nu$$

$$\Omega_m = \Omega_{bary} + \Omega_{dm}$$

$\Omega_{m,0}$	=	0.31
$\Omega_{bary,0}$	=	0.048
$\Omega_{dm,0}$	=	0.262

$\Omega_{r,0} = 9.0 \times 10^{-5}$

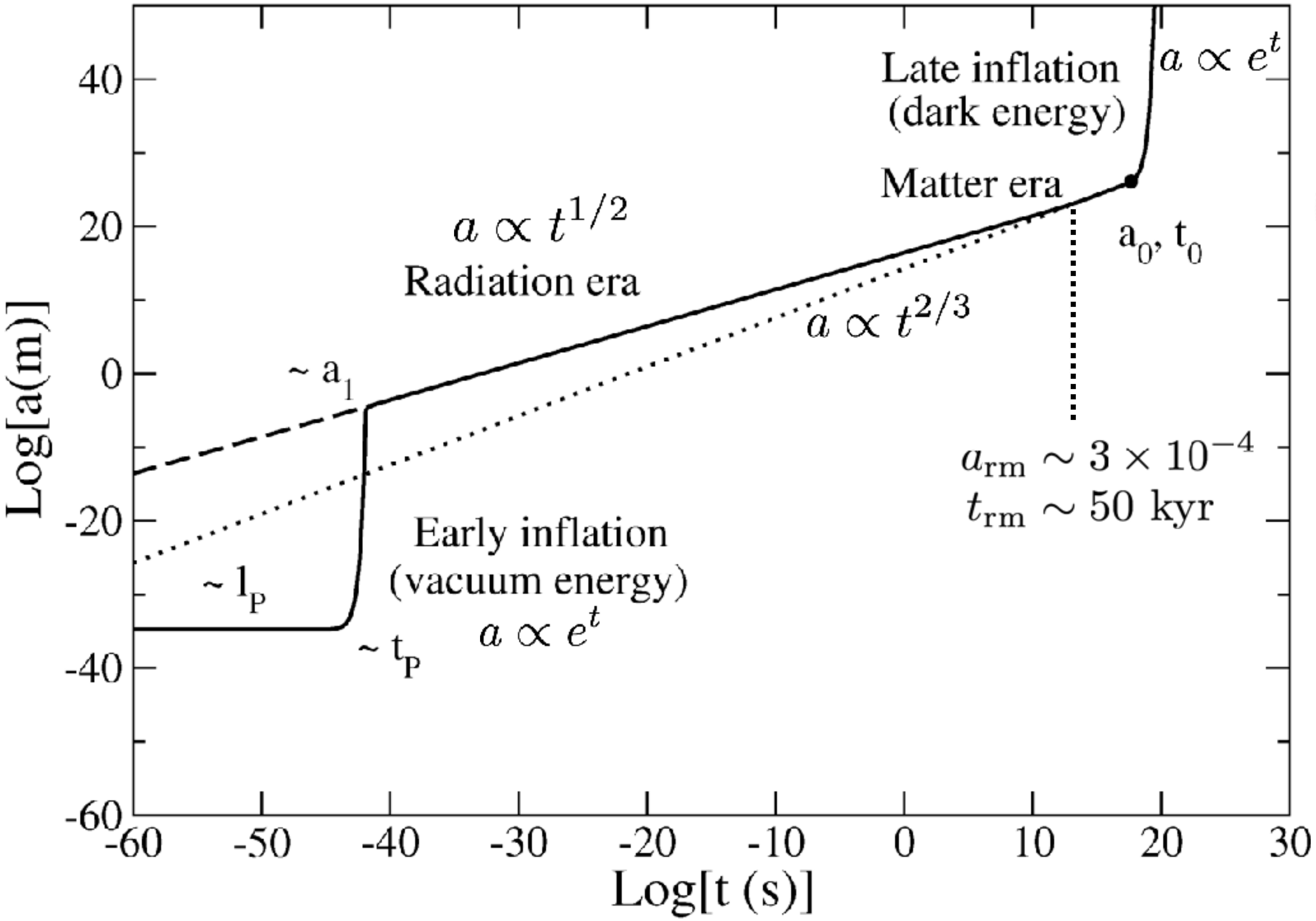
$\Omega_{\gamma,0} = 5.35 \times 10^{-5}$

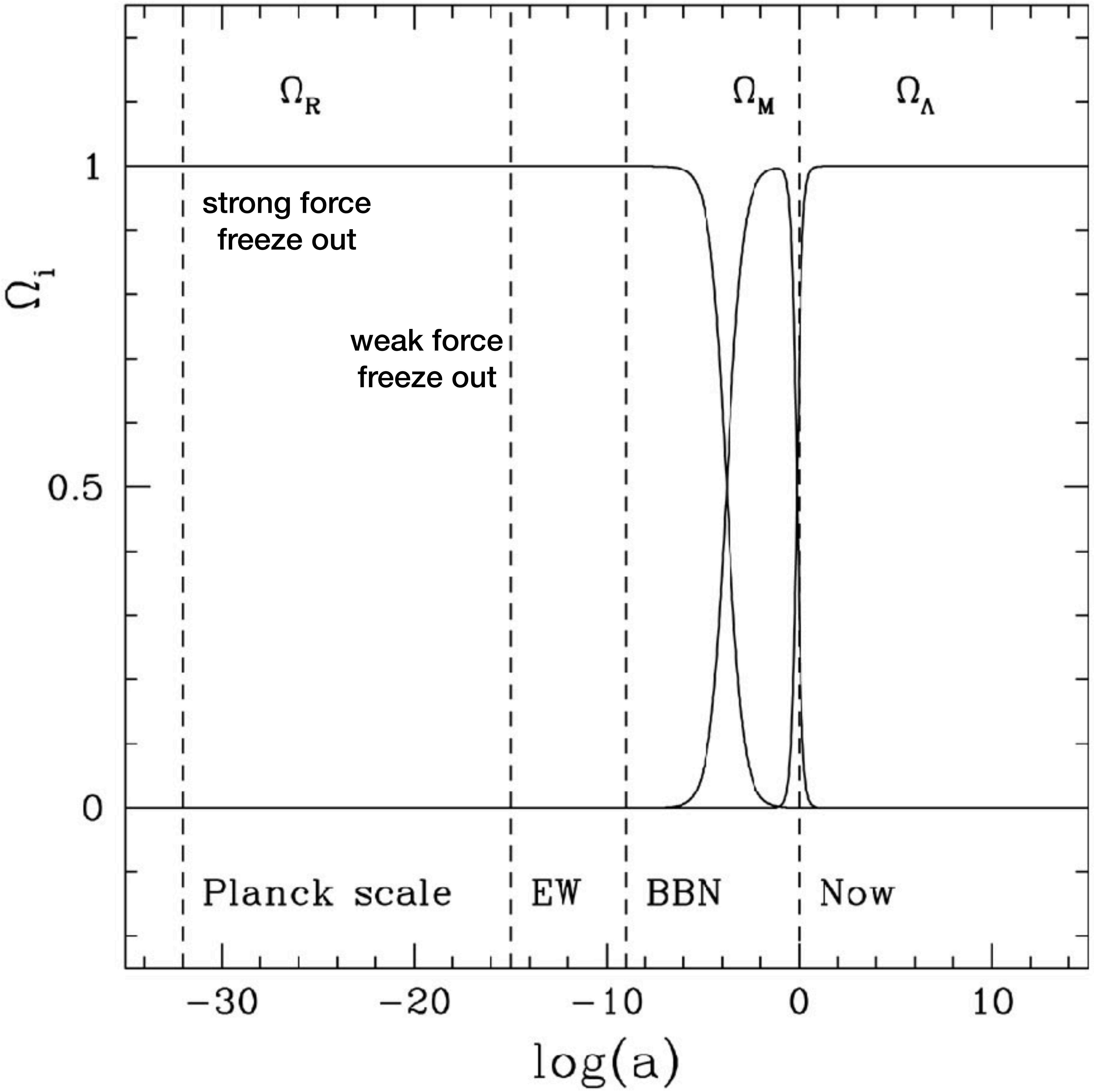
$\Omega_{\nu,0} = 3.65 \times 10^{-5}$

$$\Omega_{\Lambda,0} \approx 0.69$$

Rad. – Matter :	$a_{rm} = 2.9 \times 10^{-4}$	$t_{rm} = 50$ kyr
Matter – Λ :	$a_{m\Lambda} = 0.77$	$t_{m\Lambda} = 10.2$ Gyr
Now :	$a_0 = 1$	$t_0 = 13.7$ Gyr

Early Universe Timescales





Early Universe (Fundamental) Scales

Planck time: $t_p \equiv \left(\frac{G\hbar}{c^5}\right)^{1/2} = 5.4 \times 10^{-44}\mathrm{s}$

Planck length: $l_p \equiv \left(\frac{G\hbar}{c^3}\right)^{1/2} = 1.6 \times 10^{-33}\mathrm{cm}$

Planck mass: $M_p \equiv \left(\frac{\hbar c}{G}\right)^{1/2} = 2.2 \times 10^{-5}\mathrm{g}$

Planck energy: $E_p = M_p c^2 = \left(\frac{\hbar c^5}{G}\right)^{1/2} = 1.2 \times 10^{28}\mathrm{eV} = 1.2 \times 10^{19}\mathrm{GeV}$

Planck temperature: $T_p = E_p/k = 1.4 \times 10^{32}\mathrm{K}$

Planck units: $c = k = \hbar = G = 1$

Friedmann Eq:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2 a^2}$$

Fluid Eq:

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

Eq. of State: $P = w\varepsilon$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}\left[\varepsilon + 3P\right]$$

Key Relations

$$a = \frac{1}{1+z}$$

$$T_{\rm rad} = T_{\rm CMB,0}\, a^{-1} = 2.73\ {\rm K}\ (1+z)$$

$$\varepsilon = \sum_i \varepsilon_i$$

$$P = \sum_i w_i \varepsilon_i$$

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

$$H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2 a(t)}$$

Today, have: $H_0^2 = \frac{8\pi G}{3c^2}\varepsilon_0 - \frac{\kappa c^2}{R_0^2}$

Boundary case is $\kappa = 0$, so the critical (energy) density is

$$\varepsilon_{crit,0} = \frac{3c^2 H_0^2}{8\pi G}$$

$$\Omega(t) \equiv \frac{\varepsilon(t)}{\varepsilon_c(t)}, \quad \Omega_0 = \frac{\varepsilon(t_0)}{\varepsilon_c(t_0)}$$

$$\dot{a} = H_0 E(a)^{1/2}$$

$$E(a) = \Omega_{\mathrm{r},0} a^{-2} + \Omega\mathrm{m},0 a^{-1} + \Omega_{\Lambda,0} a^2 + (1 - \Omega_0)$$

$$t = \frac{1}{H_0}\int_0^a \frac{da}{E(a)^{1/2}}$$

$$\dot{a}^2 = \frac{8\pi G}{3c^2}\sum_i \varepsilon_{i,0} a^{-1-3w_i} - \frac{\kappa c^2}{R_0^2}$$

$w = 0$

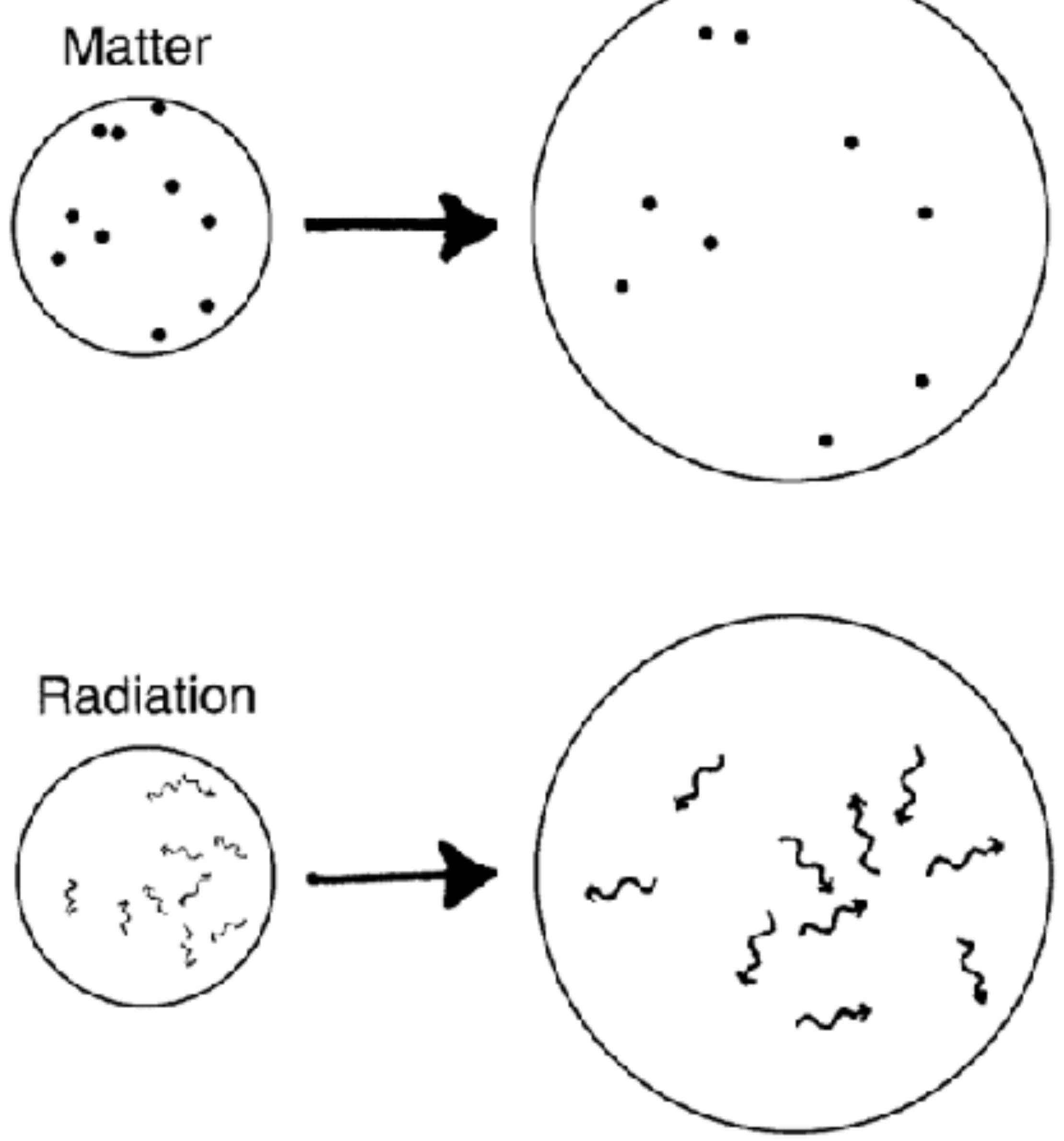
$\varepsilon_m = \varepsilon_{m,0}/a^3$

$w = \frac{1}{3}$

$\varepsilon_r = \varepsilon_{r,0}/a^4$

$w = -1$

$\varepsilon_\Lambda = \varepsilon_{\Lambda,0}$



$$d_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

$$H_0^{-1}\dot{a} = \left[\Omega_{r,0}\, a^{-2} + \Omega_{m,0}\, a^{-1} + \Omega_{\Lambda,0}\, a^2 + (1-\Omega_0)\right]^{1/2}$$

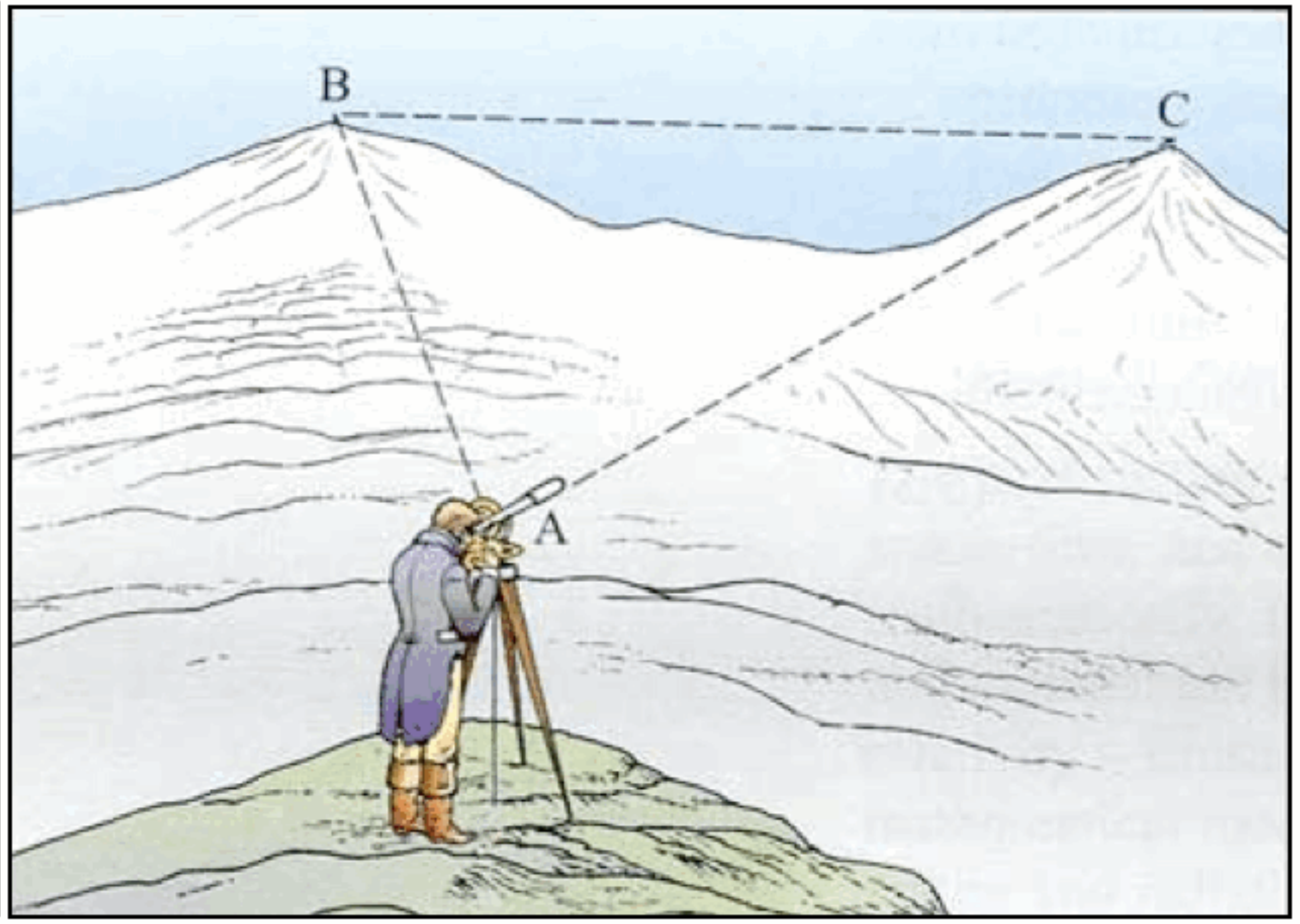
$$d_p(t_e) = \frac{1}{1+z} d_p(t_0)$$

$$t = \frac{1}{H_0}\int_0^a \frac{da}{E(a)}$$

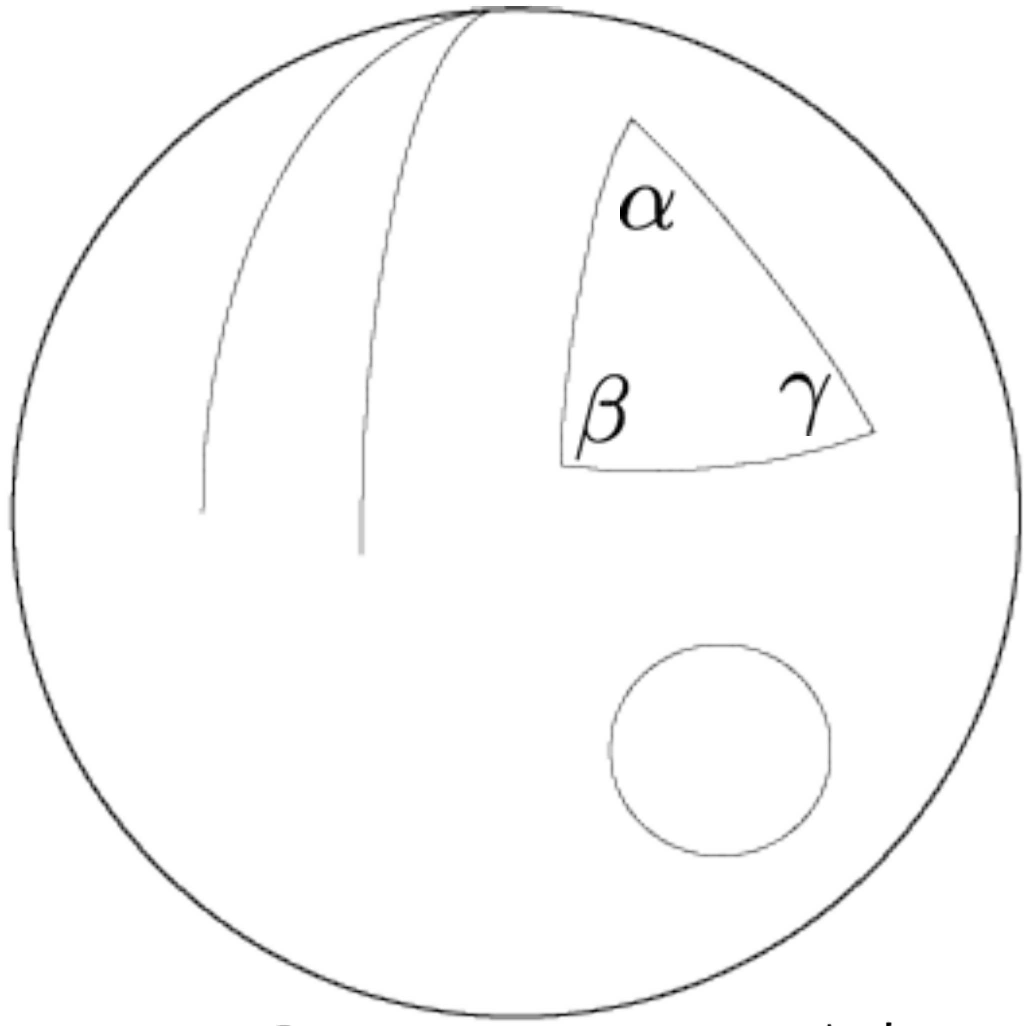
Curvature

How can we measure the curvature of spacetime?

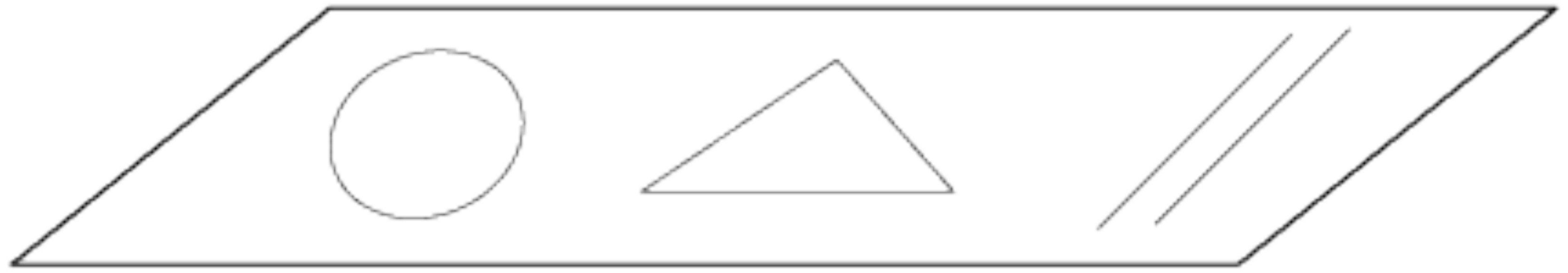
Carl Friedrich Gauss
1777 - 1855



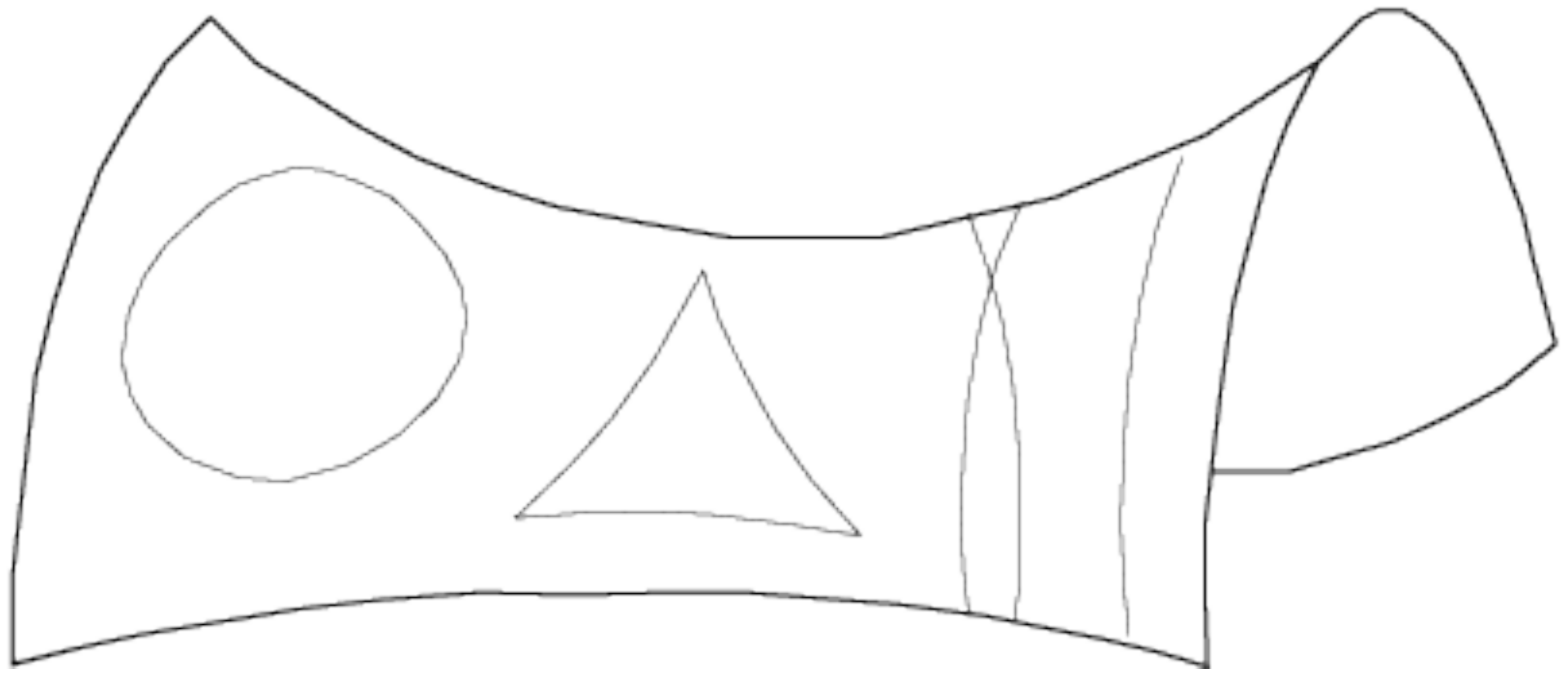
Gauss finds 180 degrees in large survey triangles:
Space is not (grossly) non-Euclidean



$\alpha + \beta + \gamma = \pi + A/R$



$\alpha + \beta + \gamma = \pi$



$\alpha + \beta + \gamma = \pi - A/R$

A = area of triangle R = Radius of Curvature

Only possible geometries that are homogeneous/isotropic

Lengths of Geodesics (3D, polar coords)

↳ straight lines in a given geometry

$$d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$$

flat or Euclidean space:

$$d\ell^2 = dr^2 + r^2 d\Omega^2$$

elliptical or spherical space:

$$d\ell^2 = dr^2 + R^2 \sin^2 \frac{r}{R} d\Omega^2$$

hyperbolic space:

$$d\ell^2 = dr^2 + R^2 \sinh^2 \frac{r}{R} d\Omega^2$$

<OR>

$$d\ell^2 = dr^2 + S_\kappa(r)^2 d\Omega^2$$

$$S_\kappa(r) = \begin{cases} R\sin\frac{r}{R} & (\kappa = +1) \\ r & (\kappa = 0) \\ R\sinh\frac{r}{R} & (\kappa = -1) \end{cases}$$

Minkowski & Robertson-Walker Metrics

metrics define the distance between events in spacetime

Minkowski (no gravity: metric in SR)

$$ds^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2$$

Robertson-Walker (with gravity, if spacetime is homogeneous & isotropic)

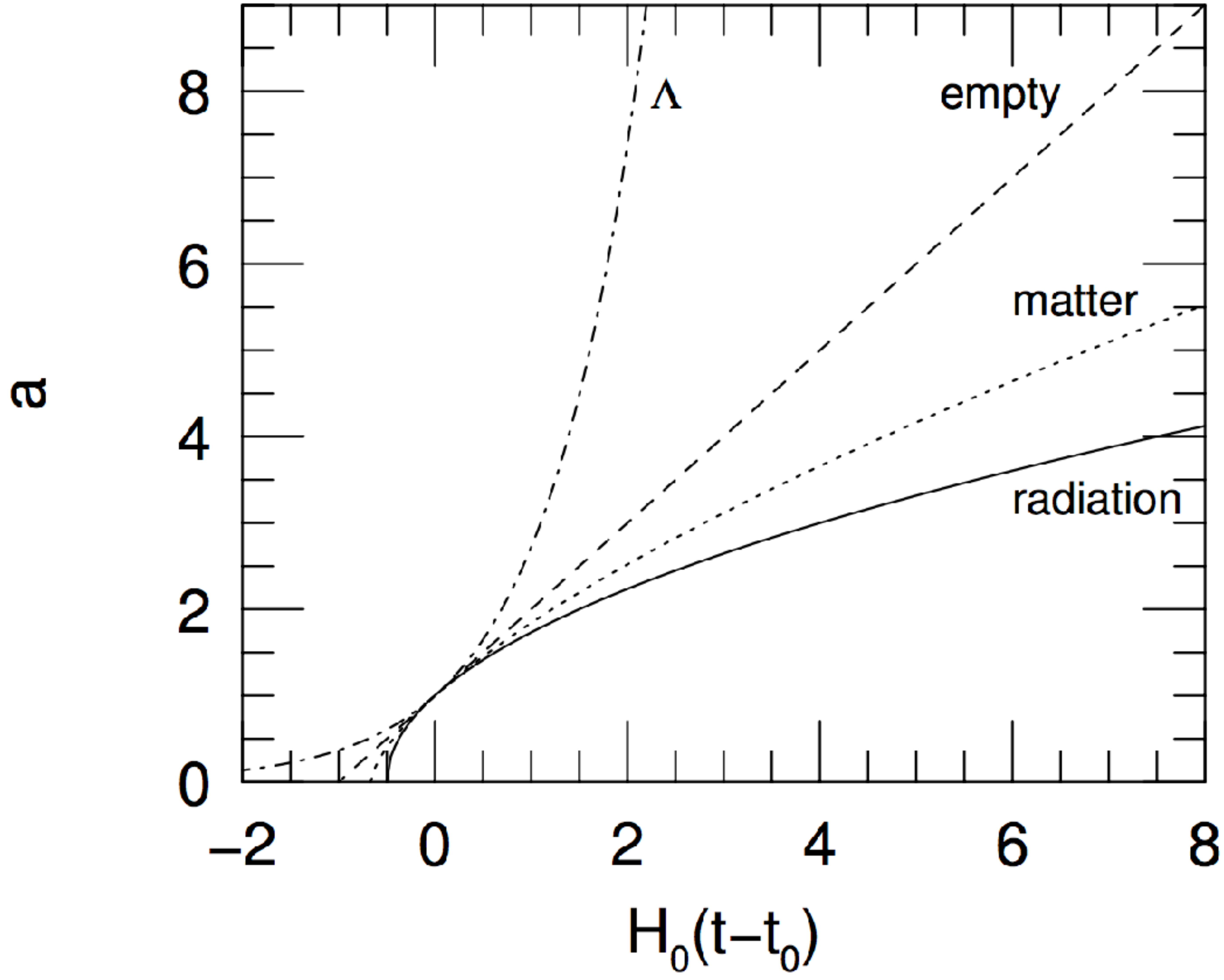
$$ds^2 = -c^2 dt^2 + a(t)[dr^2 + S_\kappa(r)^2 d\Omega^2]$$

light travels along null geodesics, i.e.: $ds^2 = 0$

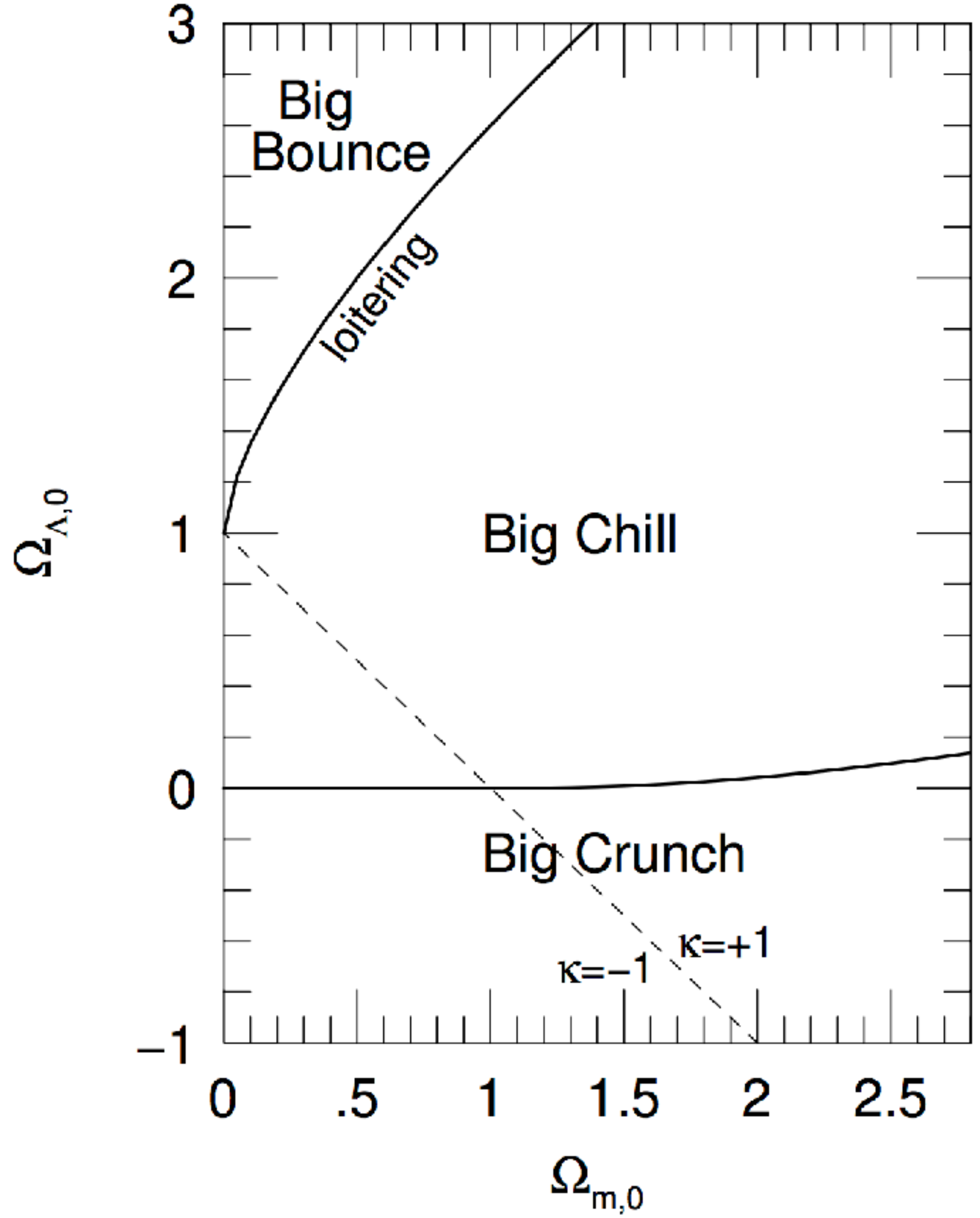
cosmological proper time or cosmic time

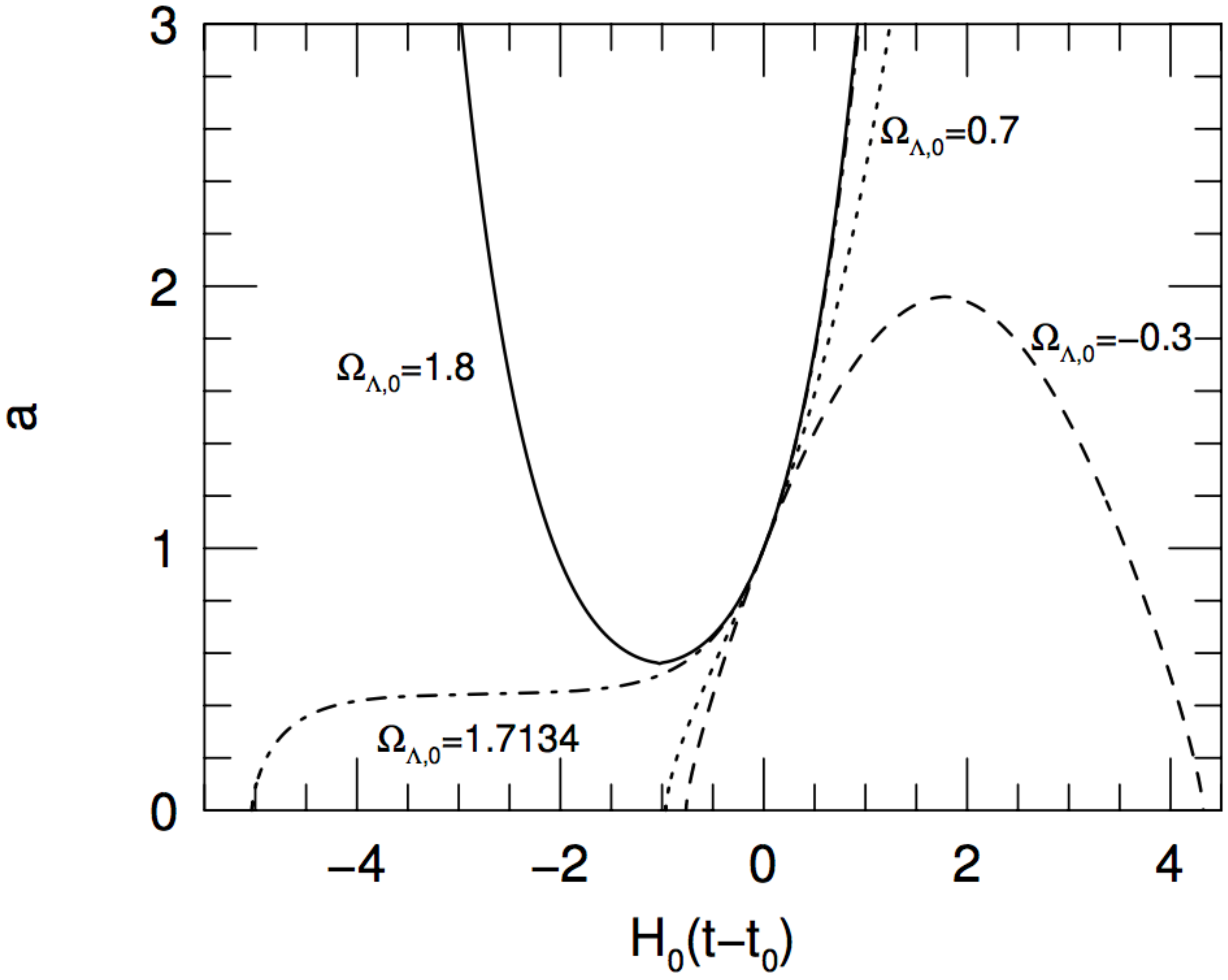
(r, θ, ϕ) comoving coordinates

Only 1 Constituent in a Flat Spacetime



Matter + Lambda + Curvature

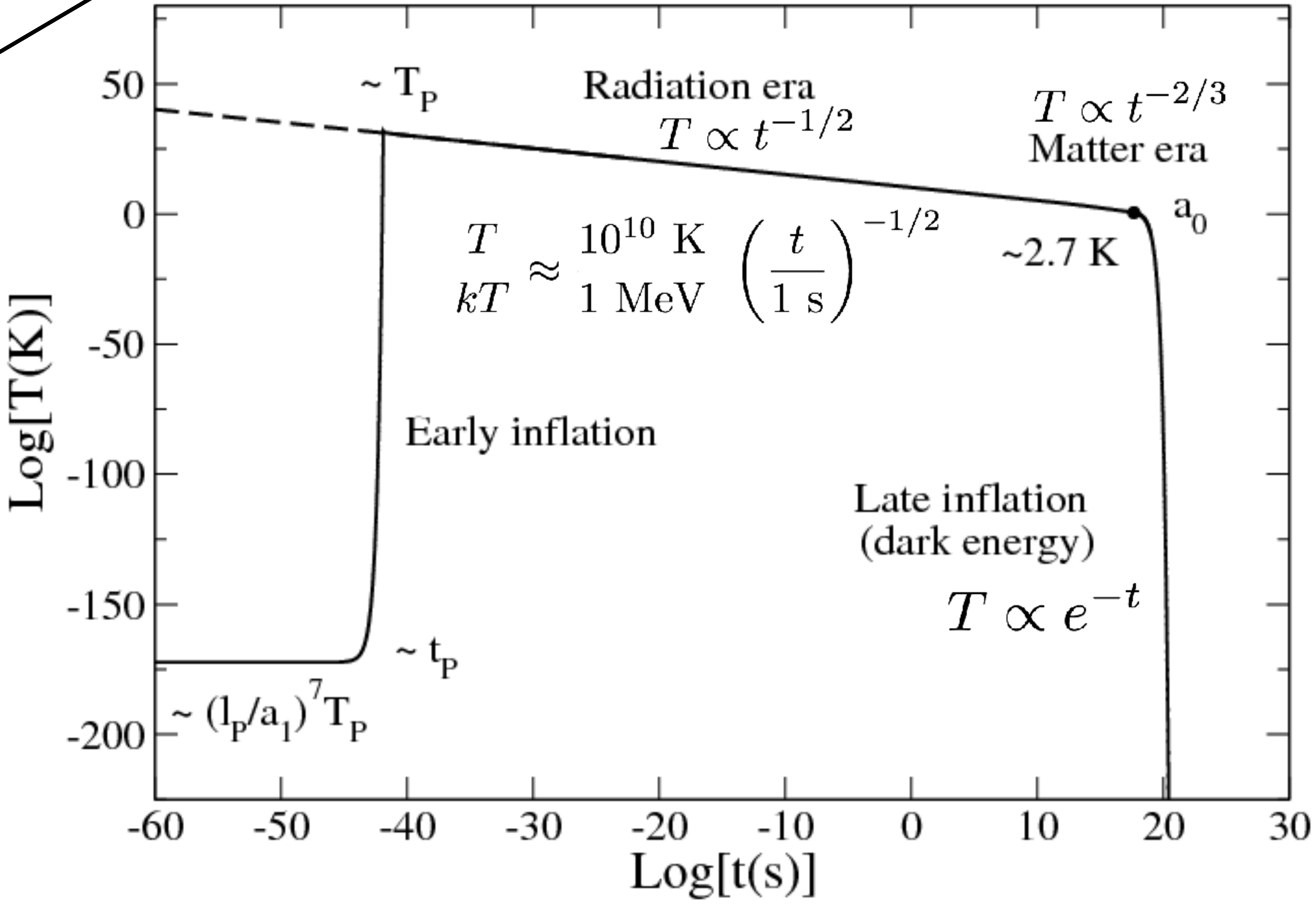




Inflation - quark soup - neutron capture - nucleosynthesis - recomb/decoup

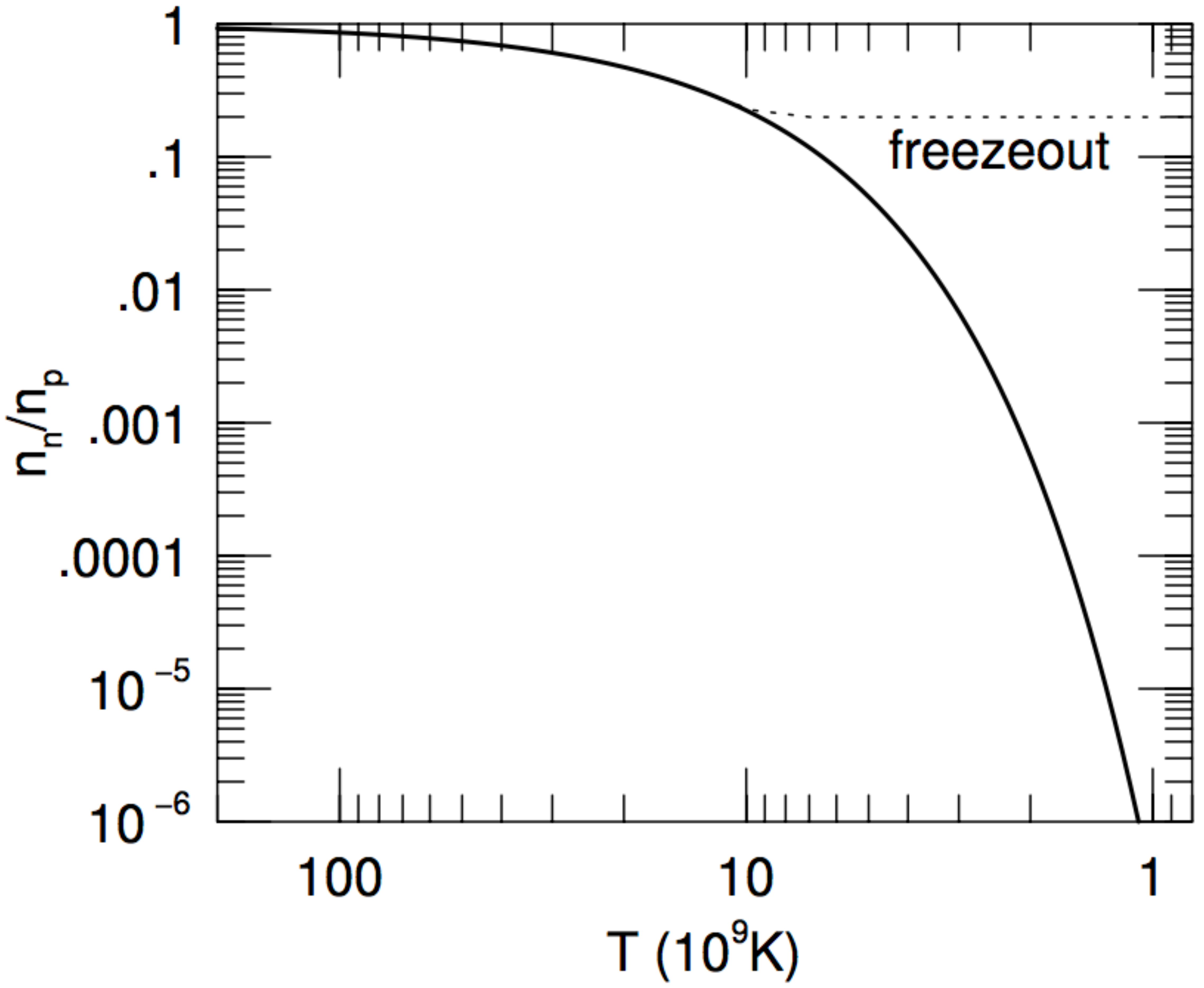
kT: 150 MeV 10 MeV 0.07 MeV 3760/2970K

baryogenesis
photon-baryon ratio



Early Universe Timescales

neutron-proton ratio

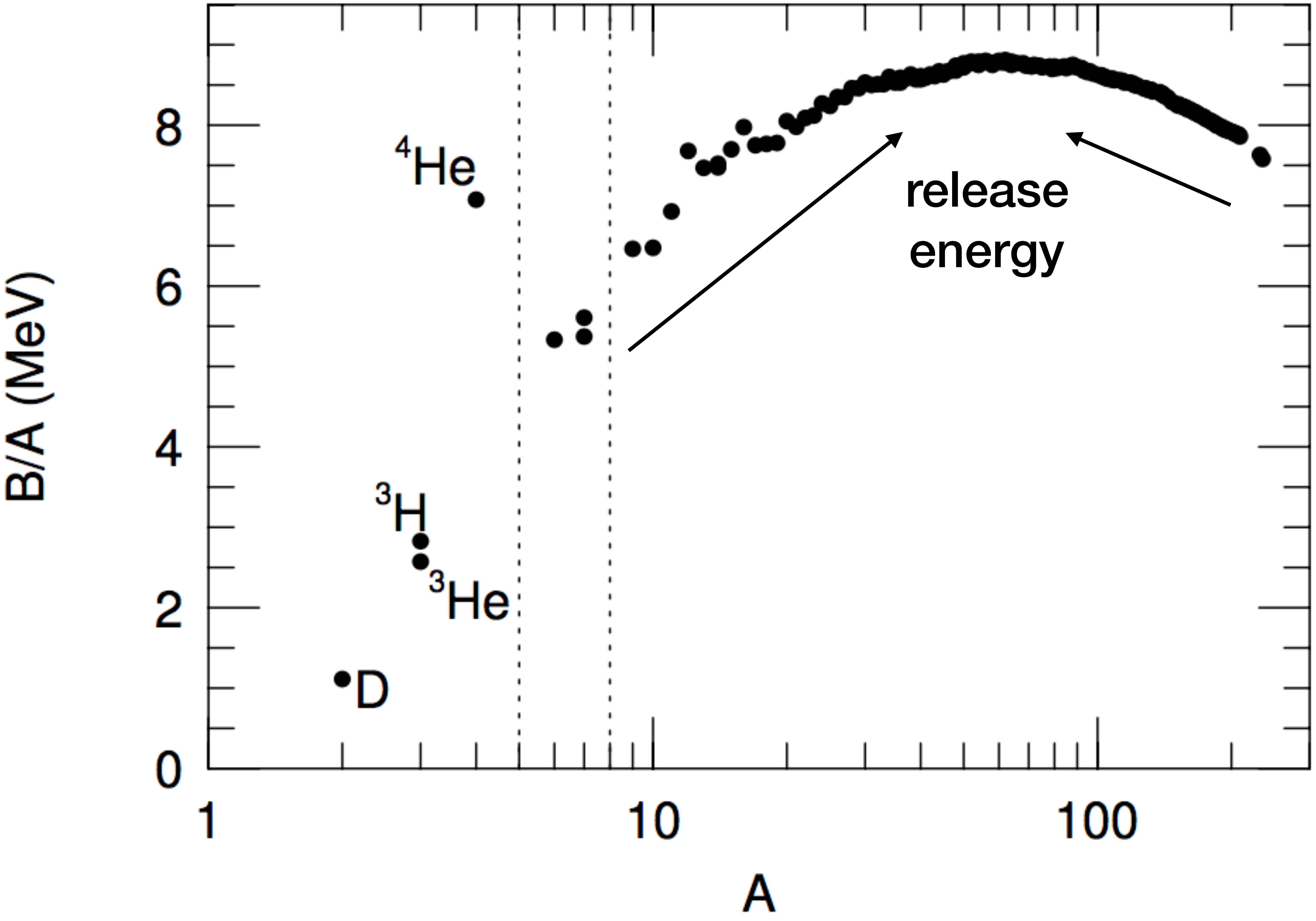


$$n_x = g_x \left(\frac{m_x kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(\frac{-m_x c^2 + \mu_x}{kT}\right)$$

$$\frac{n_n}{n_p} = \exp\left(-\frac{(m_n - m_p)c^2}{kT}\right)$$

$$\Gamma = n_\nu c \sigma_w$$

Nuclear Binding Energy



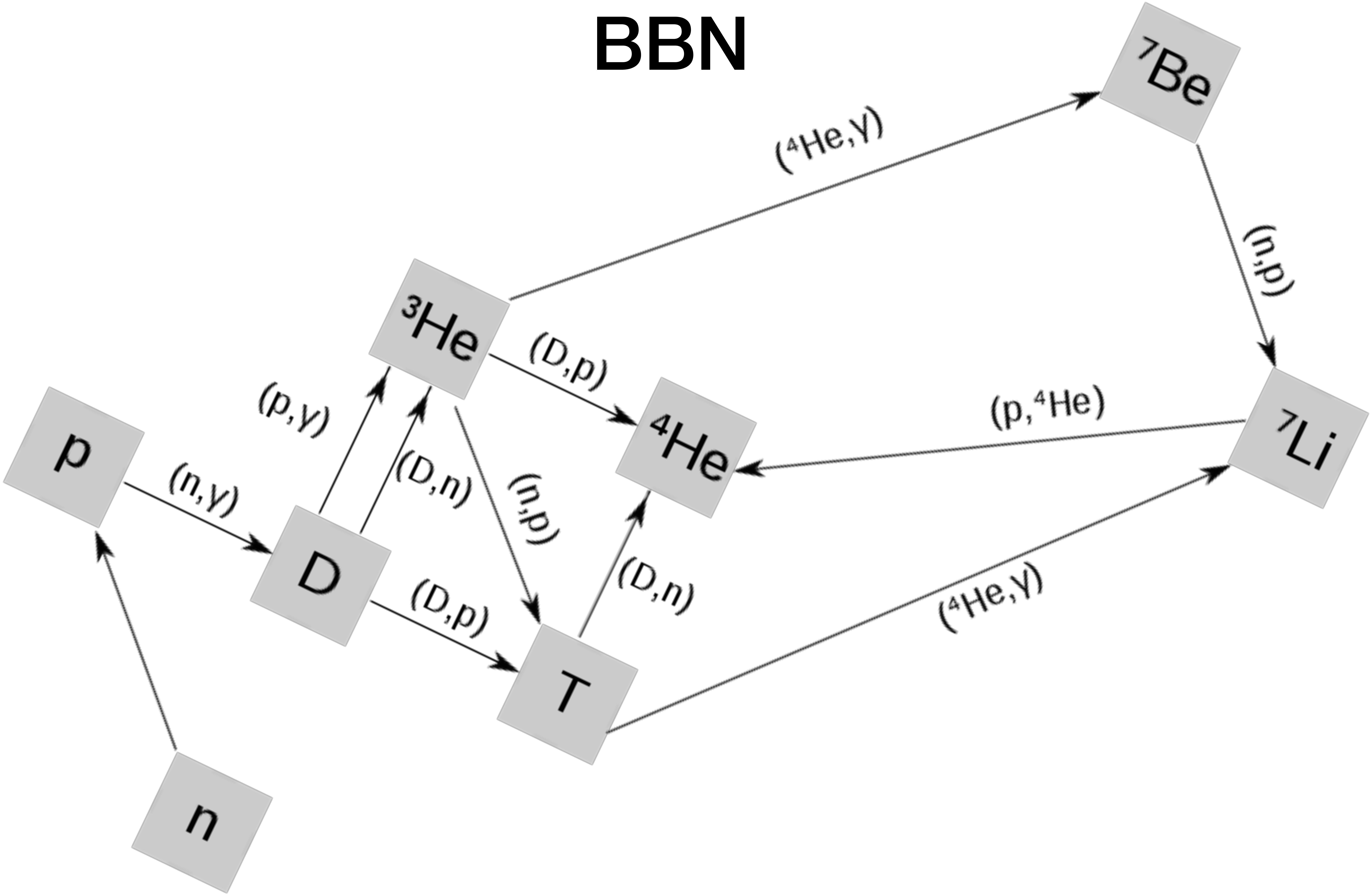
$$p + n \rightleftharpoons D + 2.22 \text{ MeV}$$

expect nucleosynthesis to result in all atoms becoming iron

does not happen - why not?

$$Y_p \equiv \frac{\rho(^4\text{He})}{\rho_{\text{bary}}}$$

BBN



Recombination $H + \gamma \rightleftharpoons p + e^-$

$$n_x(p)dp = g_x \frac{4\pi}{h^3} \frac{p^2 dp}{\exp([E - \mu_x]/kT) \pm 1}$$

(minus for bosons, plus for fermions)

g —> 2 (for non-nucleons, g_H=4)
chemical potential of photons = 0

$$\mu_H = \mu_p + \mu_e$$

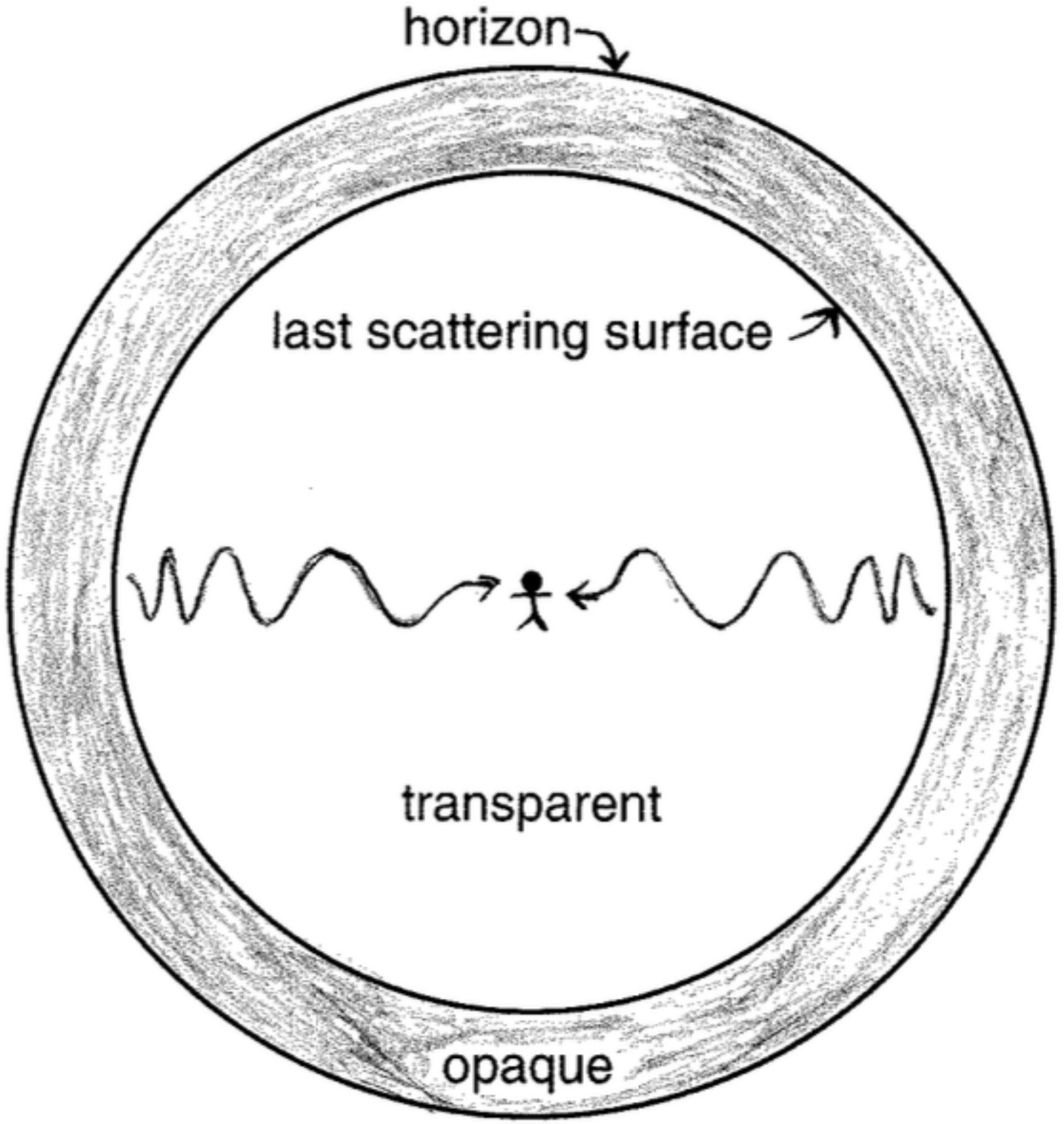
$$n_\gamma = \frac{2.4041}{\pi^2} \left(\frac{kT}{\hbar c}\right)^3$$

$$n_x = g_x \left(\frac{m_x kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(\frac{-m_x c^2 + \mu_x}{kT}\right)$$

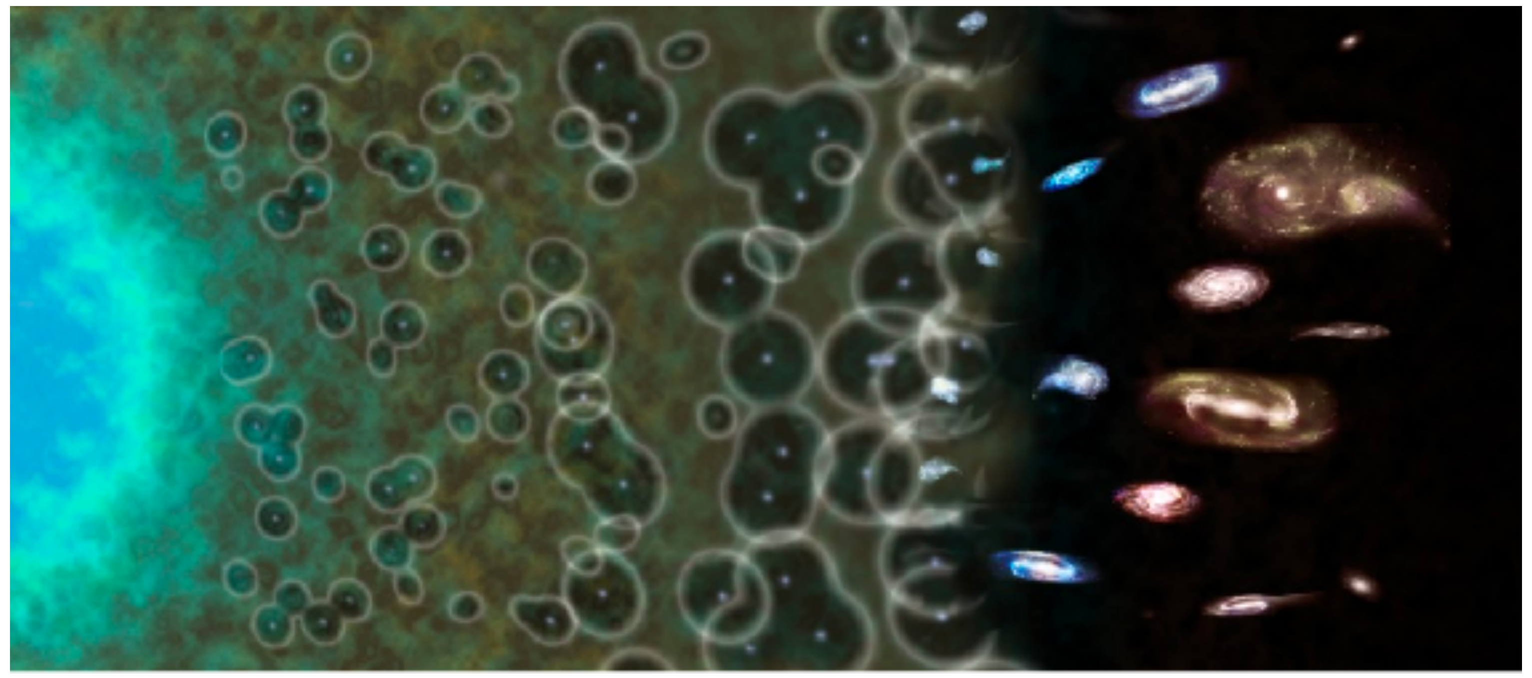
$$\frac{n_H}{n_p n_e} = \frac{g_H}{g_p g_e} \left(\frac{m_H}{m_p m_e}\right)^{3/2} \left(\frac{kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{[m_p + m_e - m_H]c^2}{kT}\right) = \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{Q}{kT}\right)$$

Saha Equation

Surface of Last Scattering



Reionization



1/15 γs get scattered out of l.o.s.

$$\tau_* = \int_{t_*}^{t_0} \Gamma(t)\,dt$$

$$\Gamma = n_e \sigma_e c$$

0.066 ± 0.0016

from Planck

(flat, matter + Λ dominated)

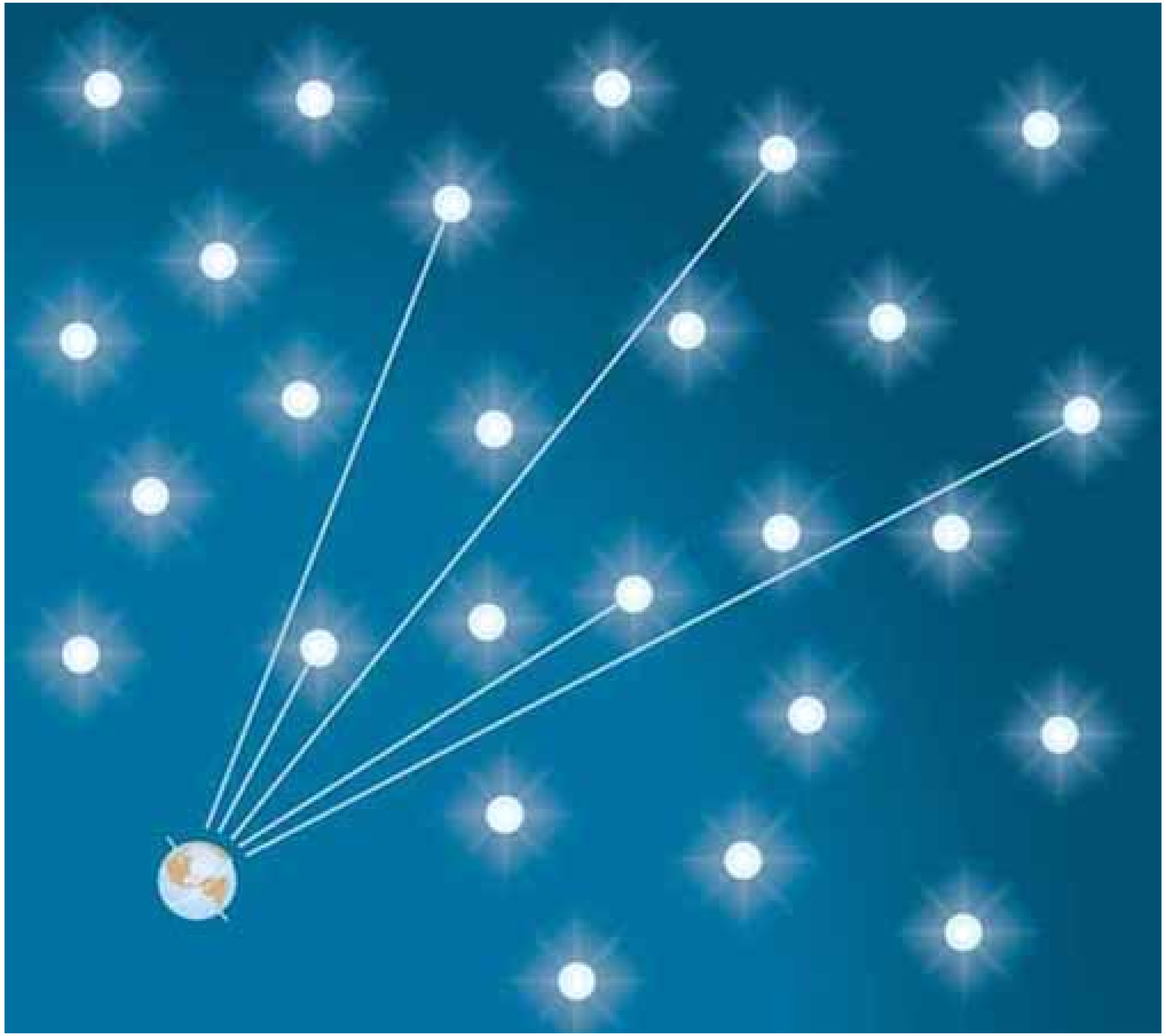
$$\tau_* = \frac{2}{3\Omega_{m,0}} \frac{\Gamma_0}{H_0} \left(\left[\Omega_{m,0}(1+z_*)^3 + \Omega_{\Lambda,0} \right]^{1/2} - 1 \right)$$

$$\boxed{z_* = 7.8 \pm 1.3}, \quad t_* = 650\,\mathrm{Myr}$$

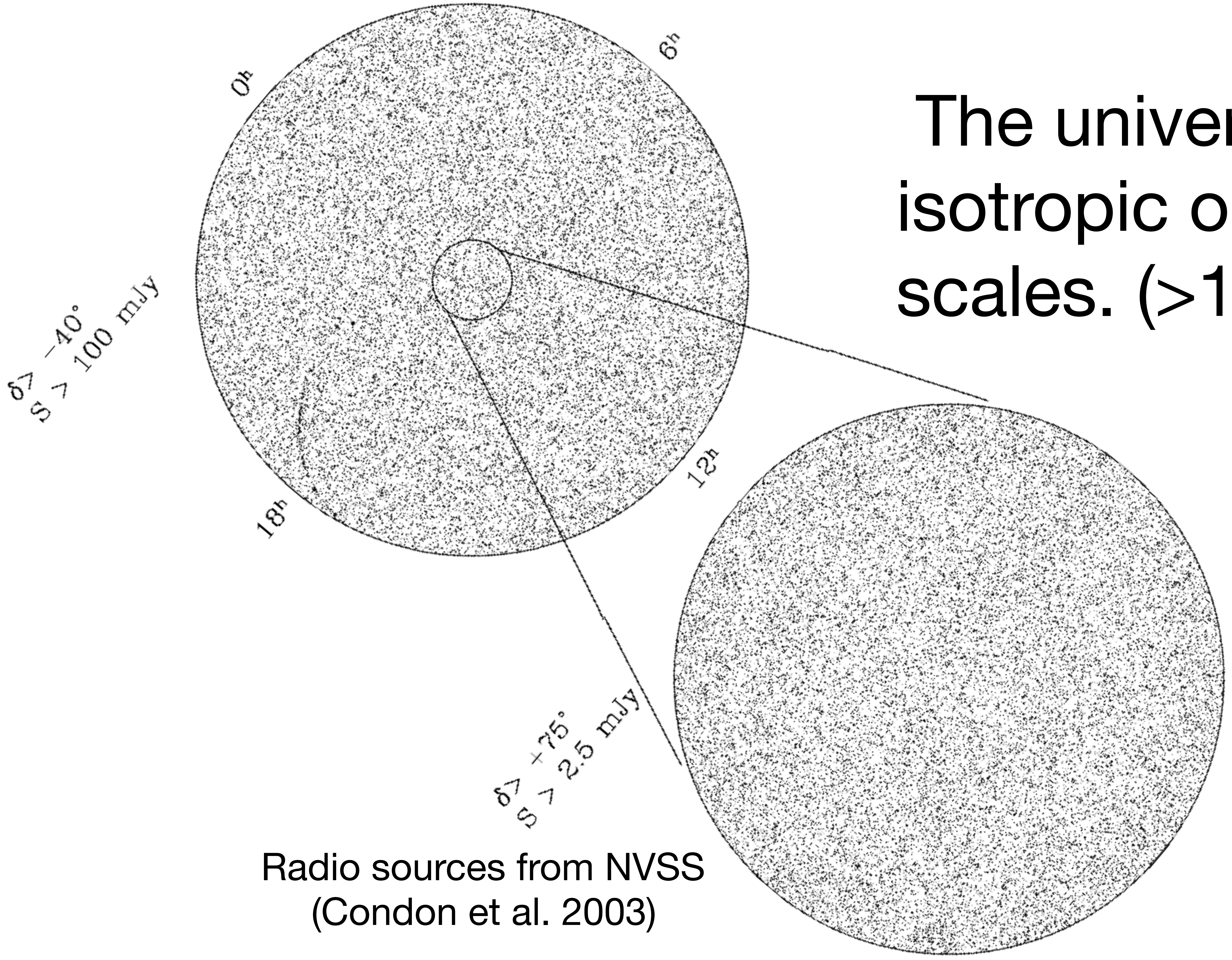
Observation

Olber's Paradox (1823)

Resolution?



Cosmological Principle



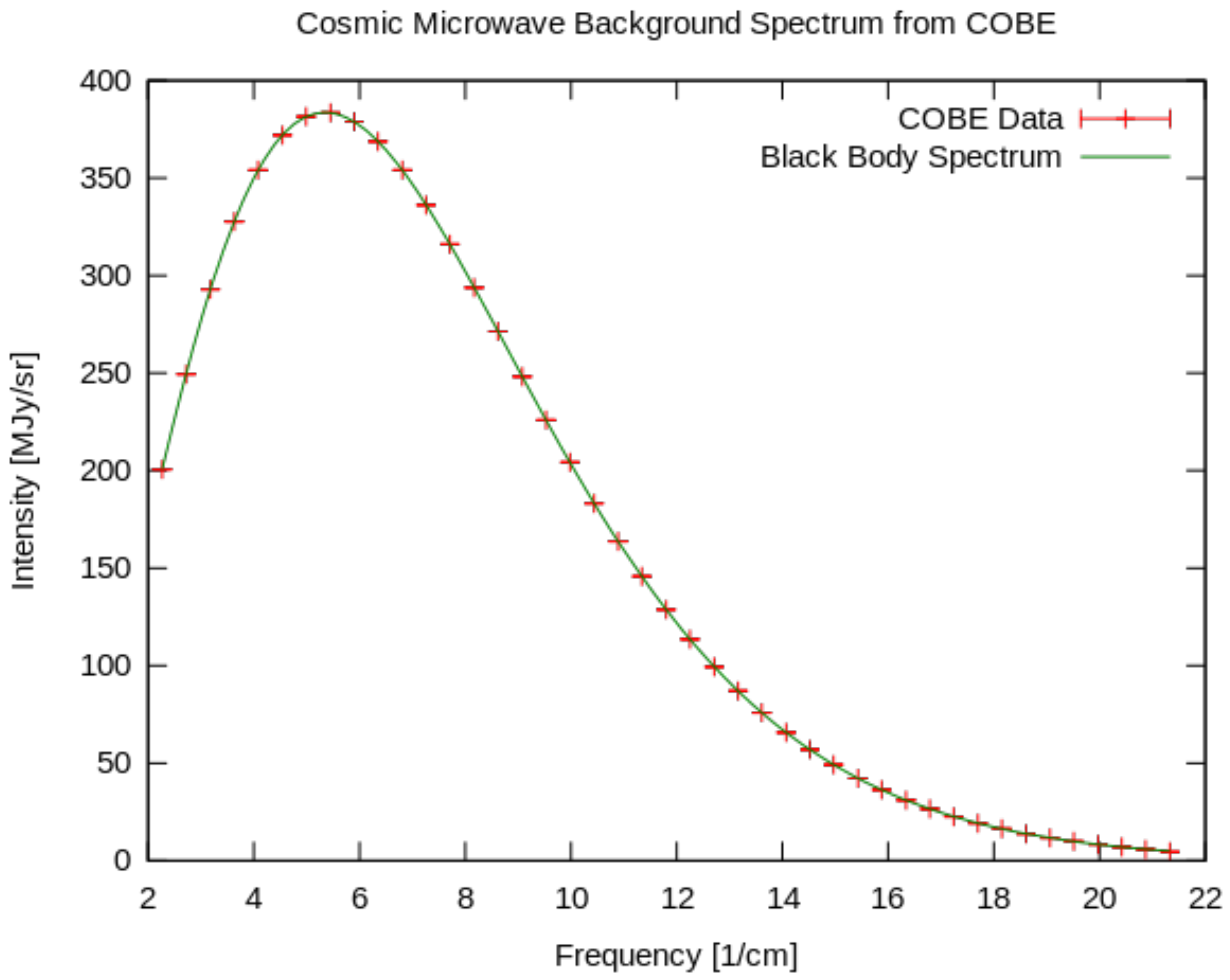
Radio sources from NVSS
(Condon et al. 2003)

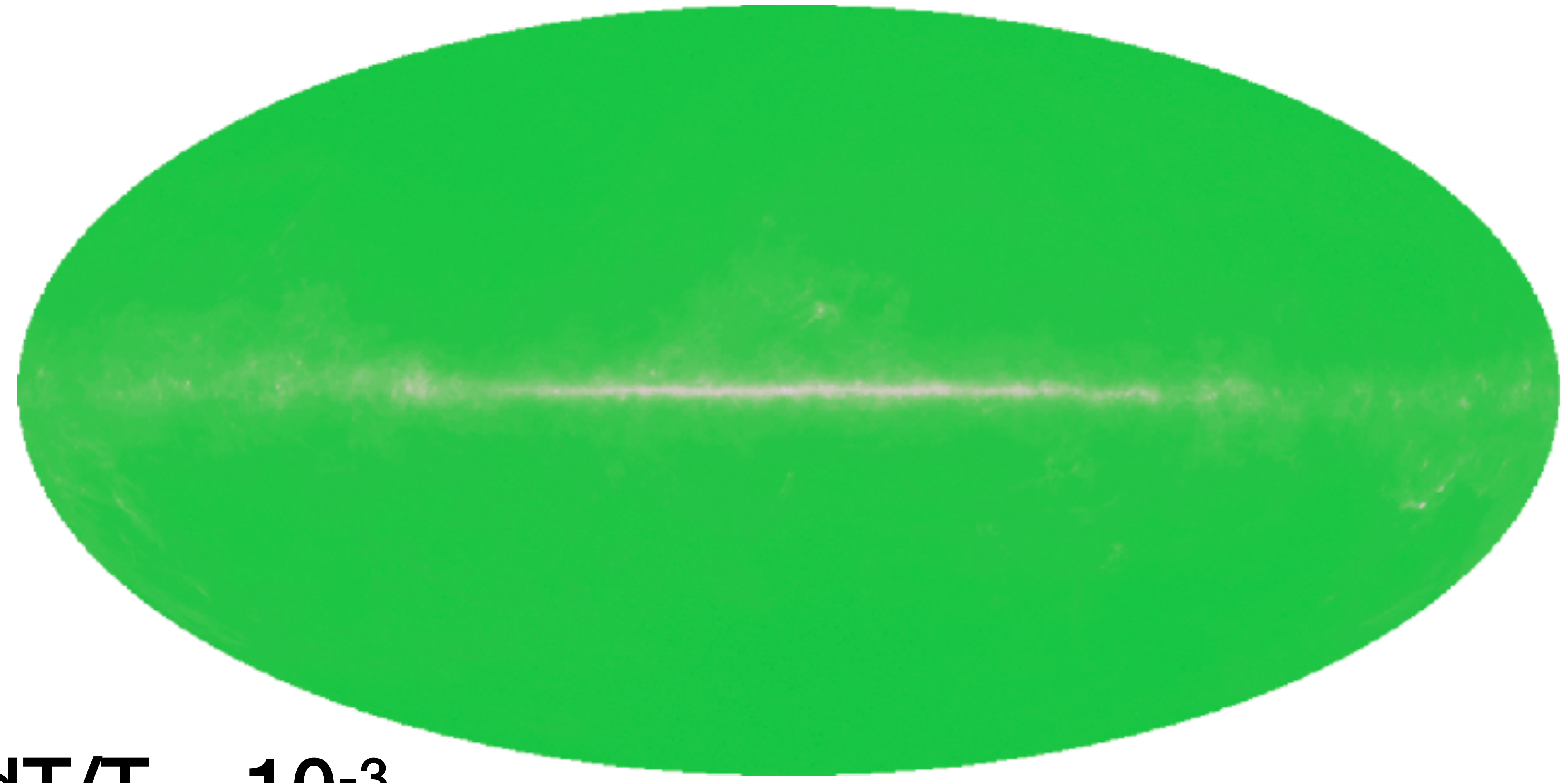
The universe is isotropic on very large scales. (>100Mpc).

Copernican Principle
=> homogeneous & isotropic

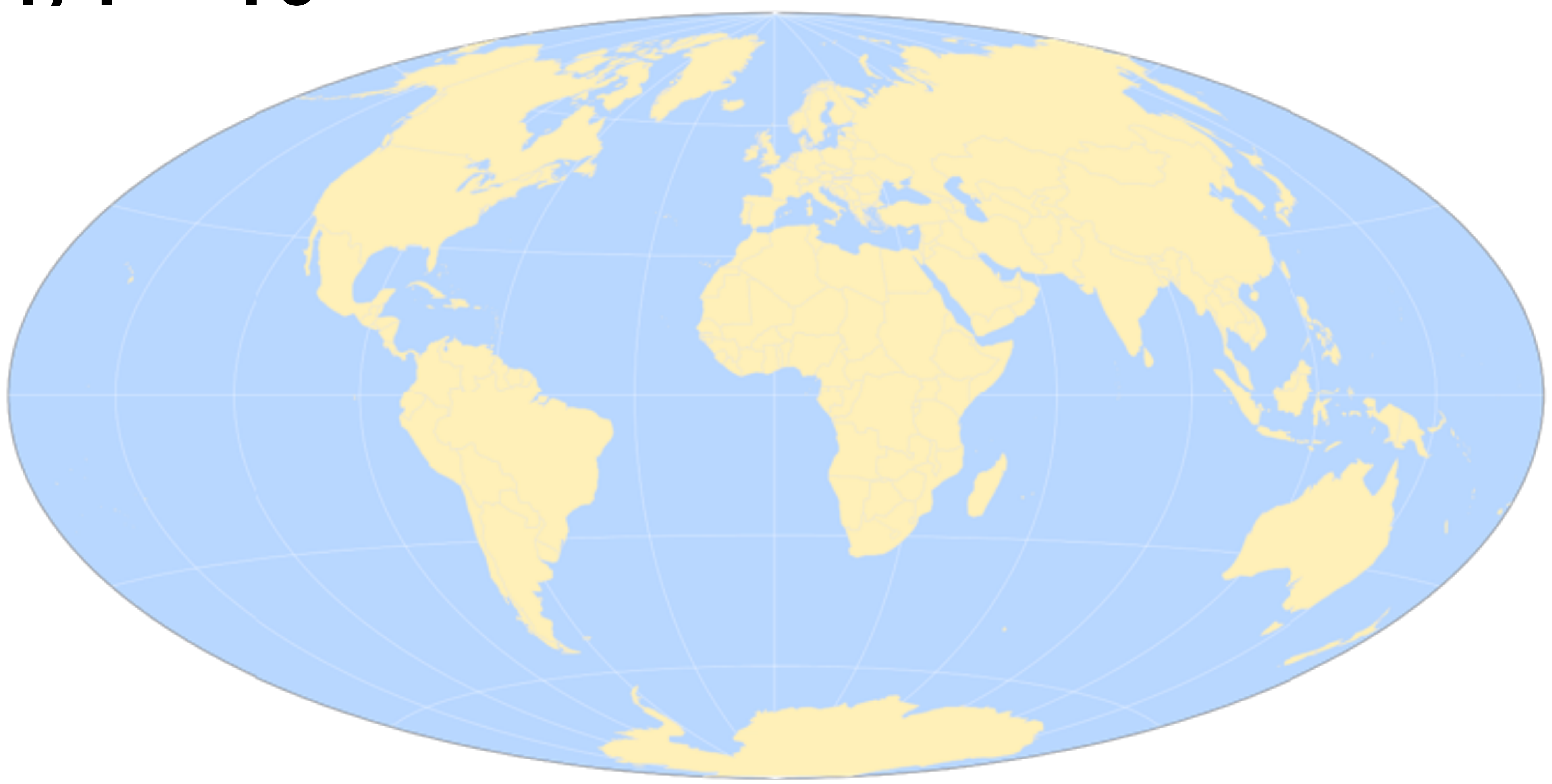
(Cosmological Principle)

Near perfect BB everywhere on the sky

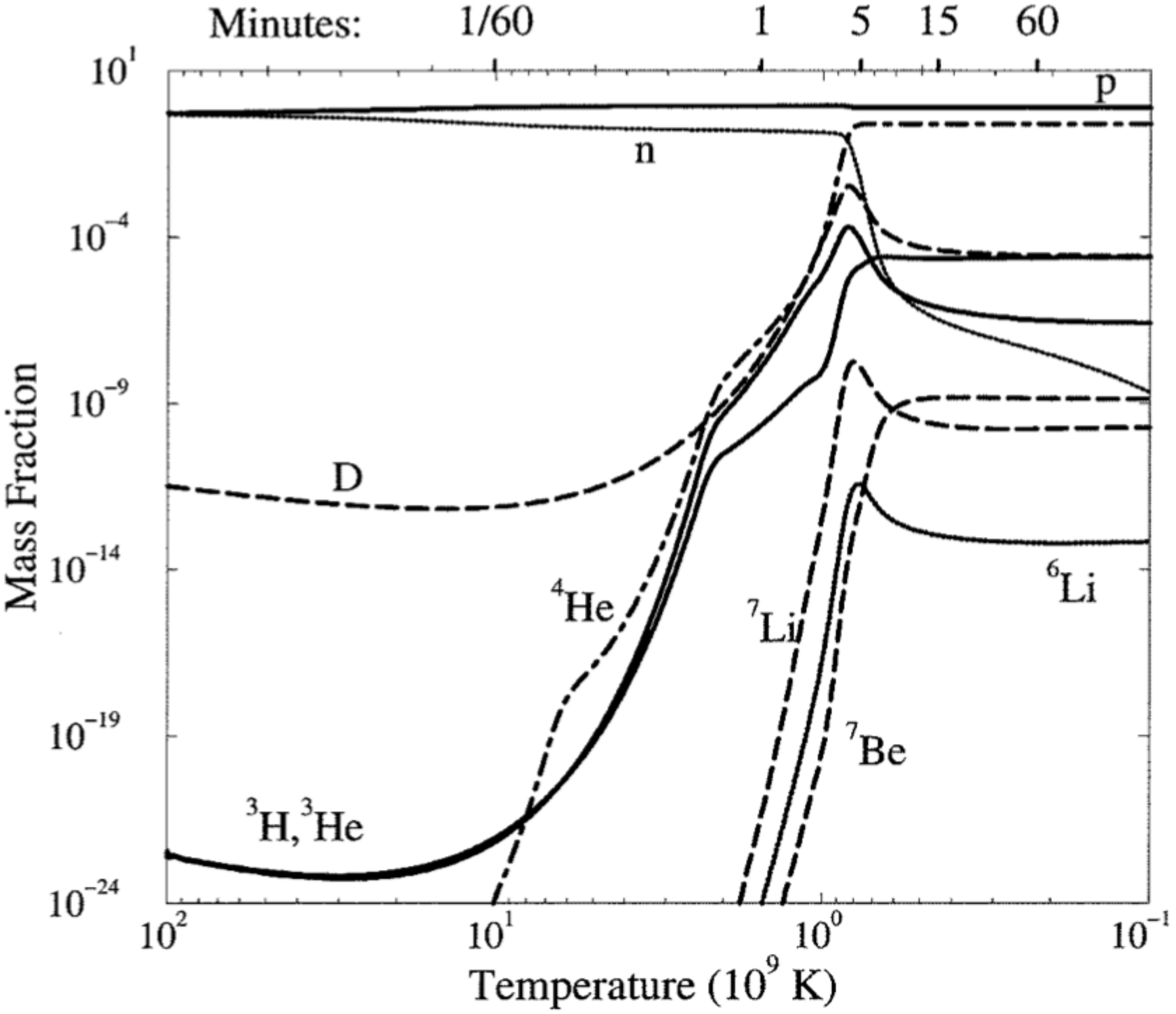




dT/T ~ 10^{-3}



Abundances from Nucleosynthesis



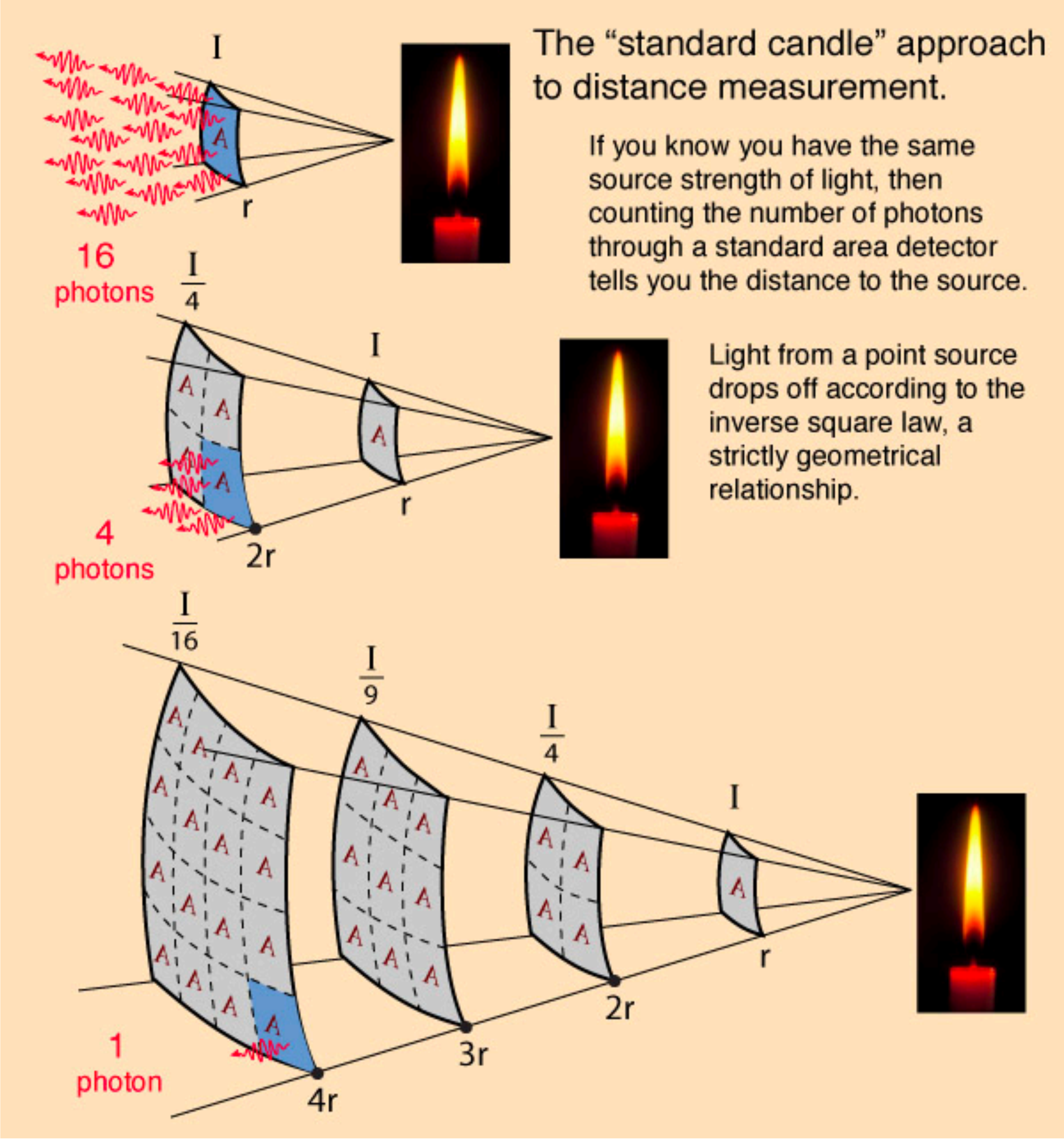
Creation process depends on relative abundances at any given time, so have to calculate computationally

Nucleosynthesis doesn't run to completion like in stars — rapidly dropping temperature cuts it off and "freezes" abundance pattern

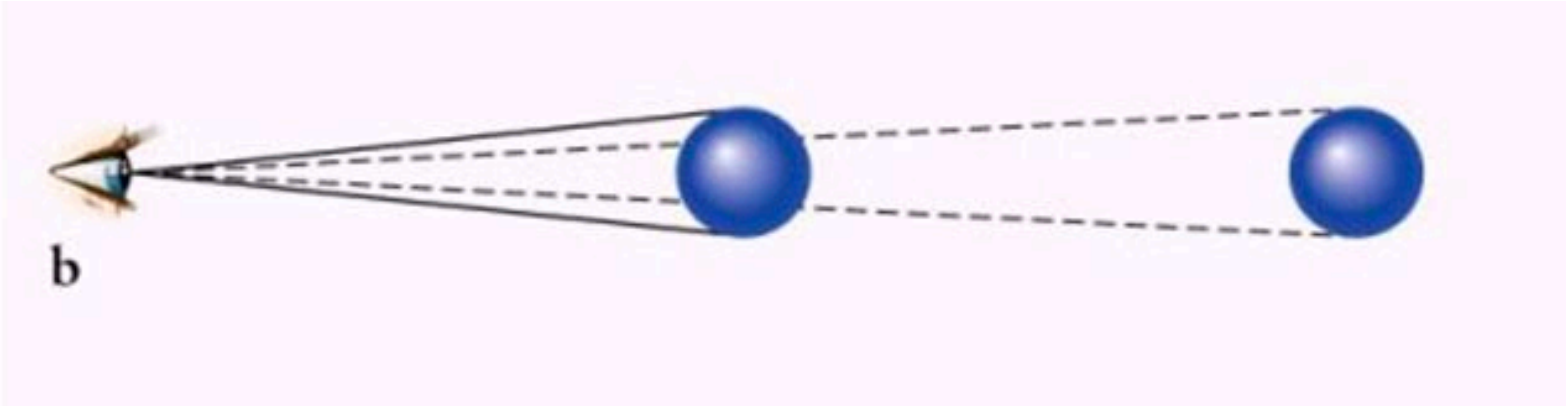
Exact yields depend most on baryon-to-photon ratio: η (determines temperature of nucleosynthesis)

Practical Distance Measures

Luminosity Distance



Angular Diameter Distance



Practical Distance Measures

$$d_L = \sqrt{\frac{L}{4\pi f}}$$

$$d_A = \frac{D}{\theta}$$

in flat, static universe,

$$d_L = d_A = d_p$$

$$d_p \rightarrow ds^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2$$

$$ds^2 = -c^2 dt^2 + a^2 \left[dr^2 + S_K(r)^2 d\Omega^2 \right]$$

$$S_K \begin{cases} R_0 \sin r/R_0 & K = +1 \\ r & K = 0 \\ R_0 \sinh r/R_0 & K = -1 \end{cases}$$

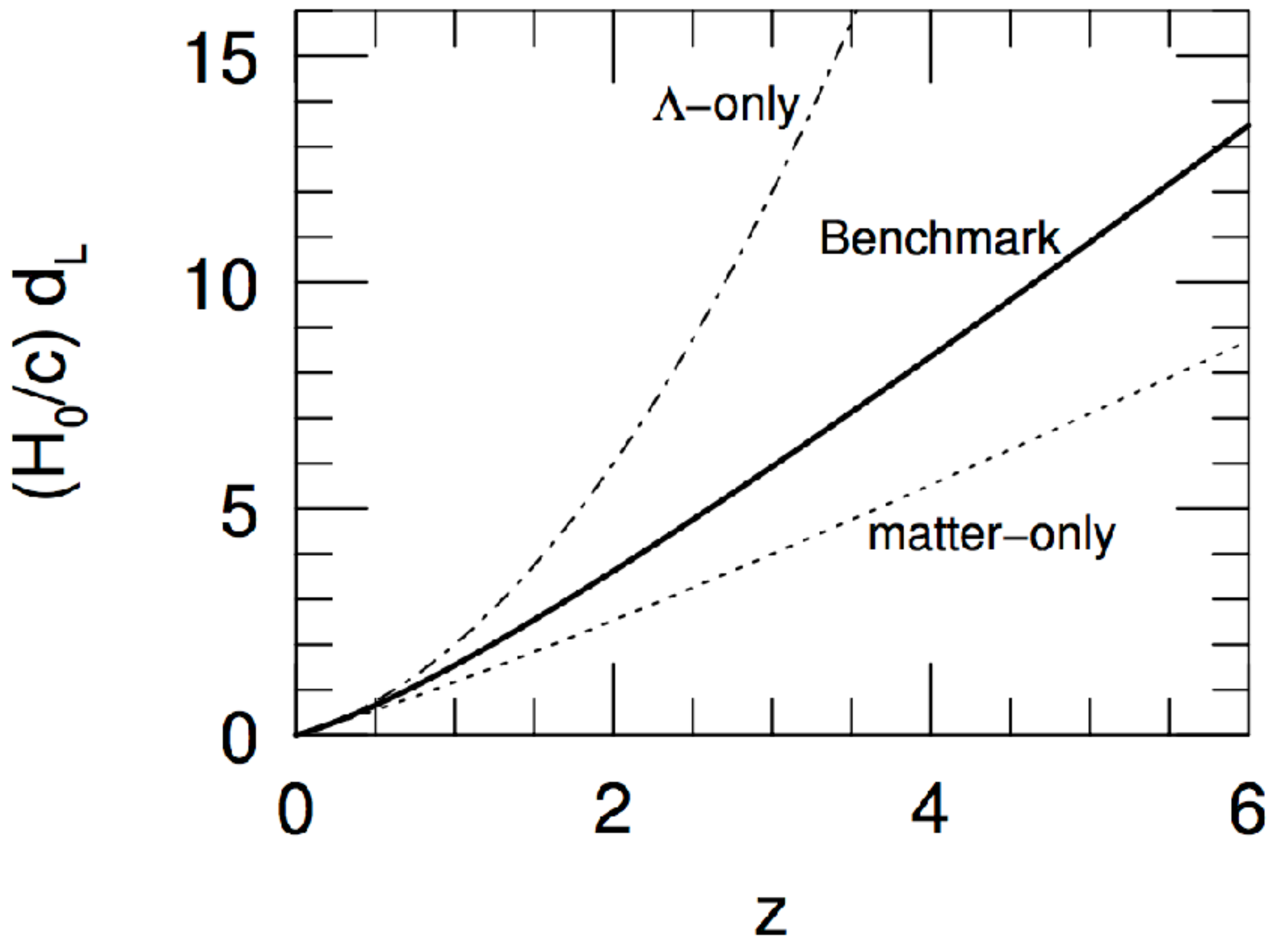
$$\frac{S_K(r)}{1+z} = d_A$$

$$d_L = S_K(r)(1+z)$$

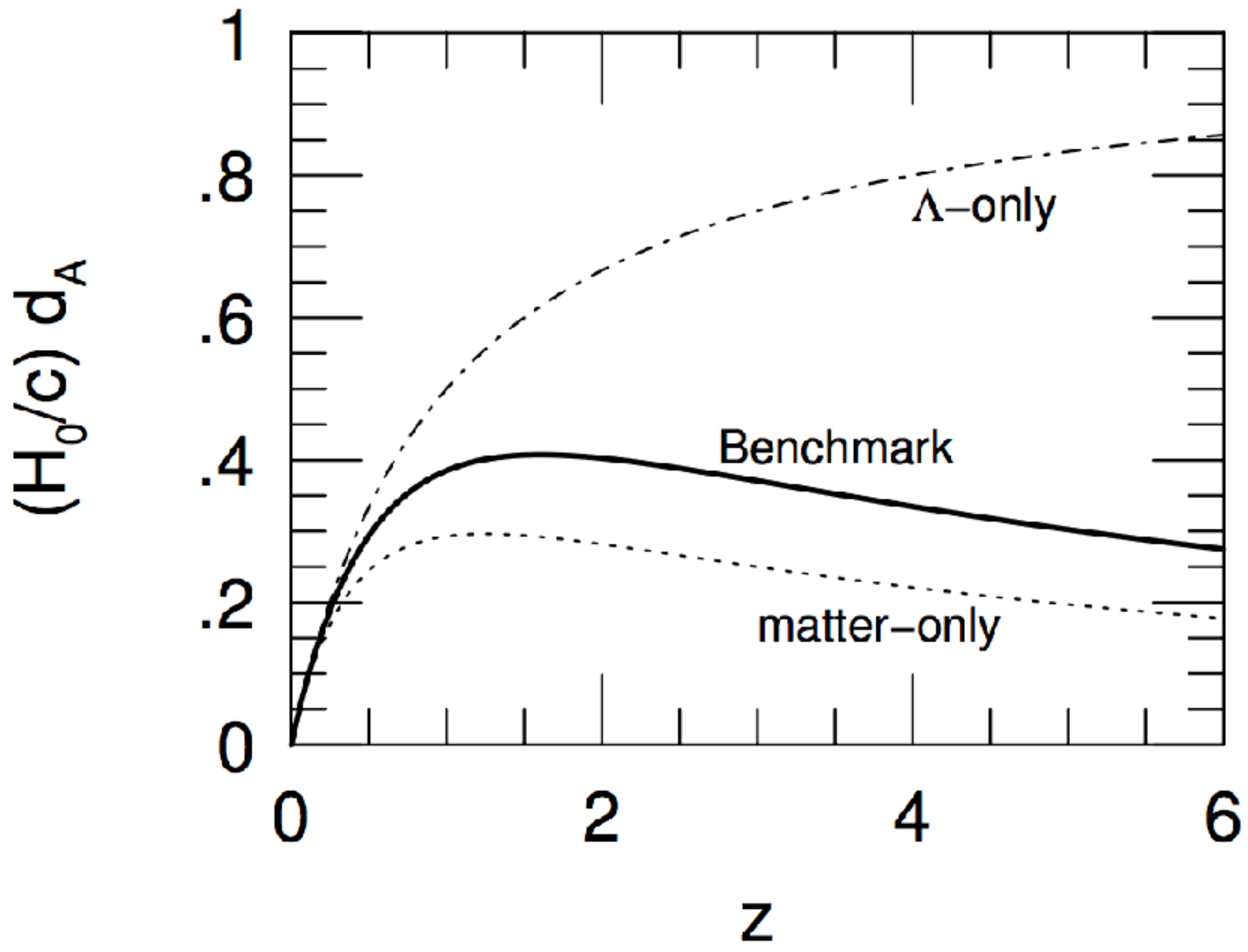
$$K = 0, \quad d_A = \frac{d_p(t_0)}{1+z} = \frac{d_L}{(1+z)^2}$$

How distances are affected by underlying cosmology

Luminosity Distance



Angular Diameter Distance



Practical Distance Measures

can define

$$q_0 = -\frac{\ddot{a}a}{\dot{a}^2}\bigg|_{t=t_0} = -\frac{\ddot{a}}{aH^2}\bigg|_{t=t_0}$$

$$a(t) \approx 1 + H_0(t-t_0) - \frac{1}{2}q_0 H_0^2 (t-t_0)^2$$

$$d_p(t_0) \approx \frac{c}{H_0}\left[z - \left(1+\frac{q_0}{2}\right)z^2\right] + \frac{cH_0}{2}\frac{z^2}{H_0} = \frac{cz}{H_0}\left[1 - \frac{1+q_0}{2}z\right]$$

$$q_0 = -\frac{\ddot{a}}{aH^2}\bigg|_{t=t_0} = \frac{1}{2}\sum \Omega_{i,0}(1+3w_i)$$

$$d_L \cong \frac{cz}{H_0}\left(1 + \frac{1-q_0}{2}z\right)$$

For the general case (rad, matter, Λ), set

$$q_0 = \Omega_{r,0} + \frac{1}{2}\Omega_{m,0} - \Omega_{\Lambda,0}$$

$$d_A = \frac{cz}{H_0}\left(1 - \frac{3+q_0}{2}z\right)$$

Getting distances to the nebulae

$$v_p(t_0) \equiv H_0 d_p(t_0) \rightarrow d_H(t_0) \equiv c/H_0$$

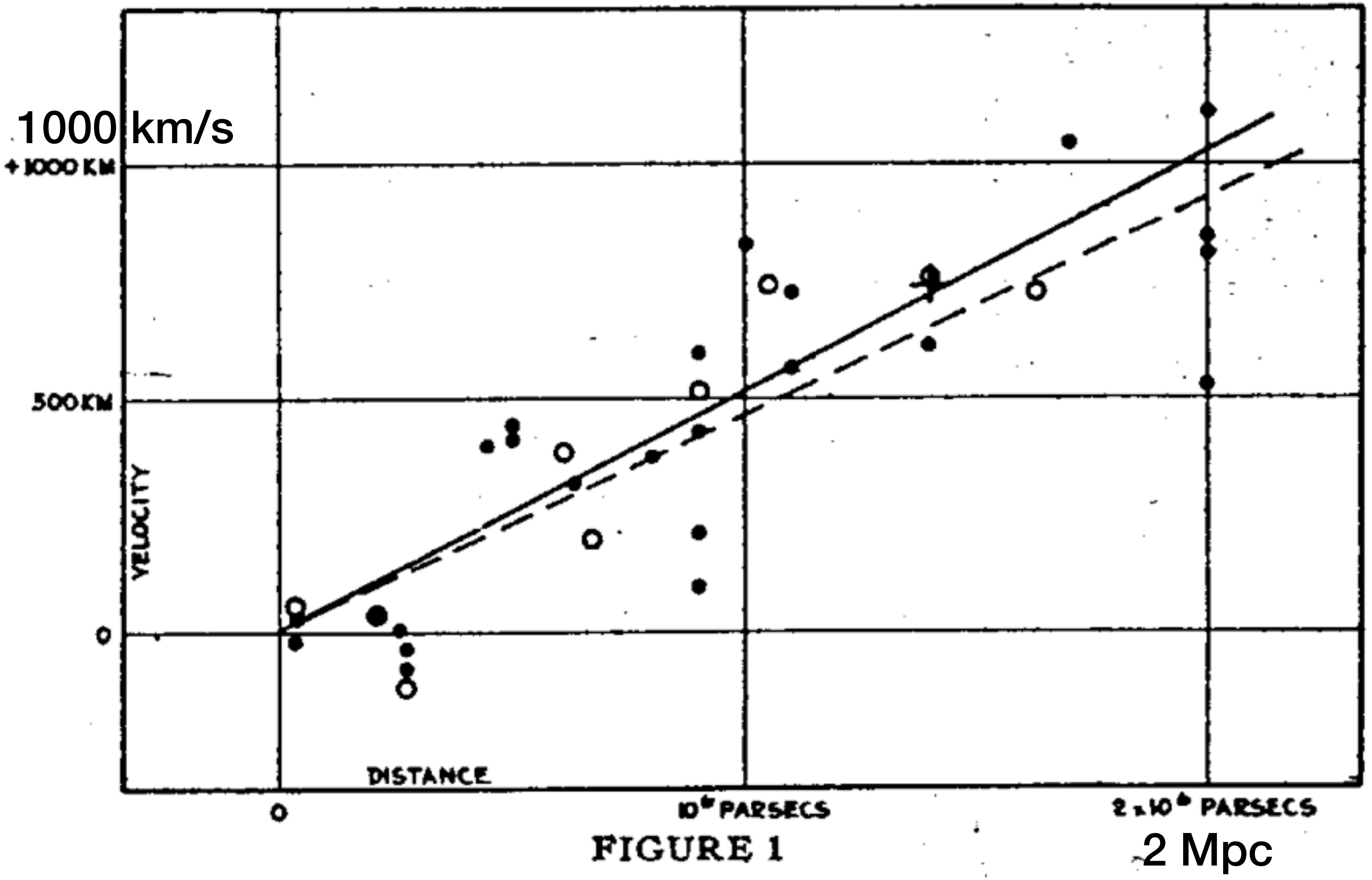


FIGURE 1

Practical Distance Measures

$$m \equiv -2.5 \log_{10}(f/f_{ref})$$

$$f_{ref} = 2.53\times10^{-8}\ \mathrm{W\,m^{-2}}$$

$$M \equiv -2.5 \log_{10}(L/L_{ref})$$

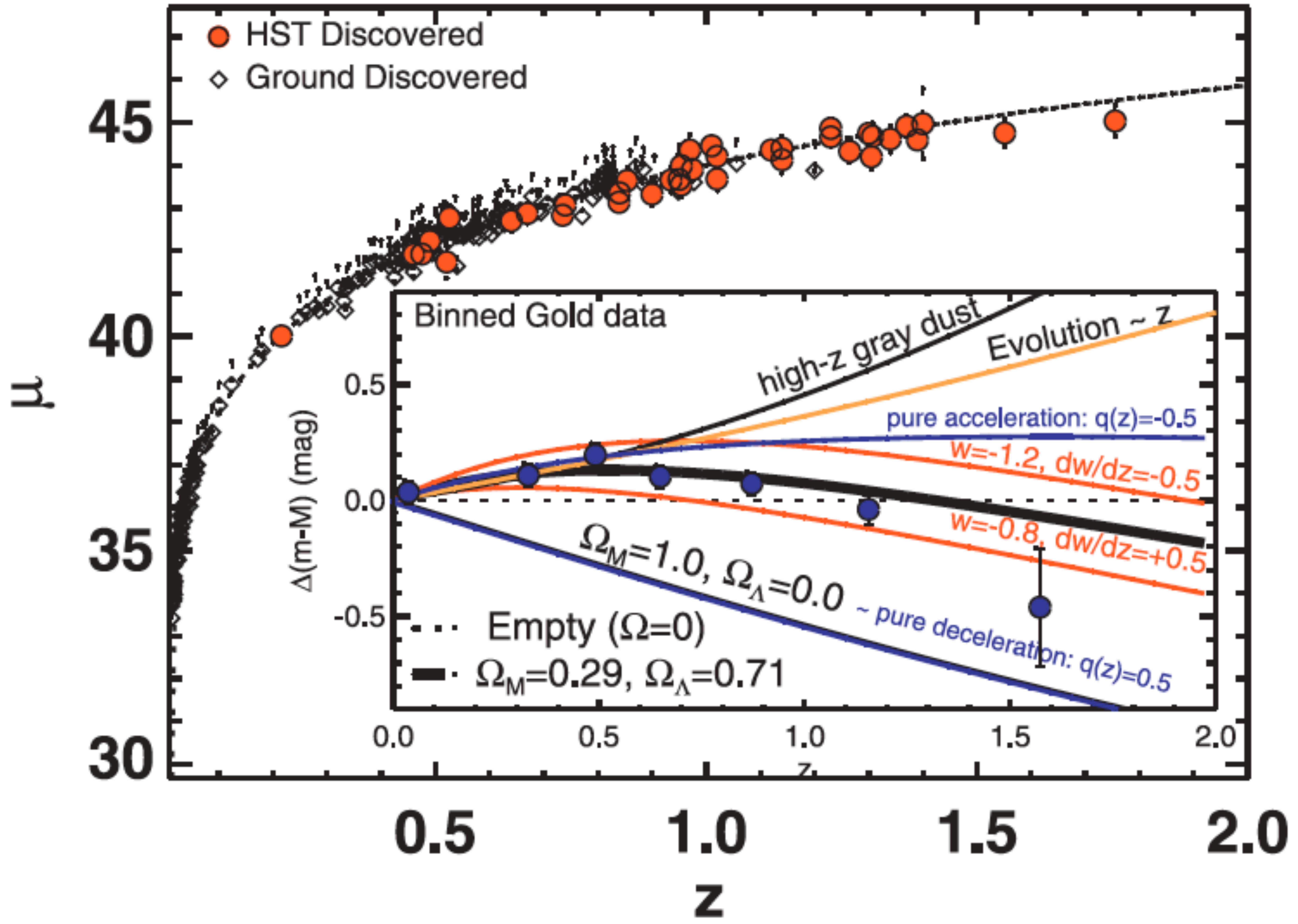
$$L_{ref} = 78.7\ L_\odot$$

→ $M \Rightarrow$ value of m @ 10pc

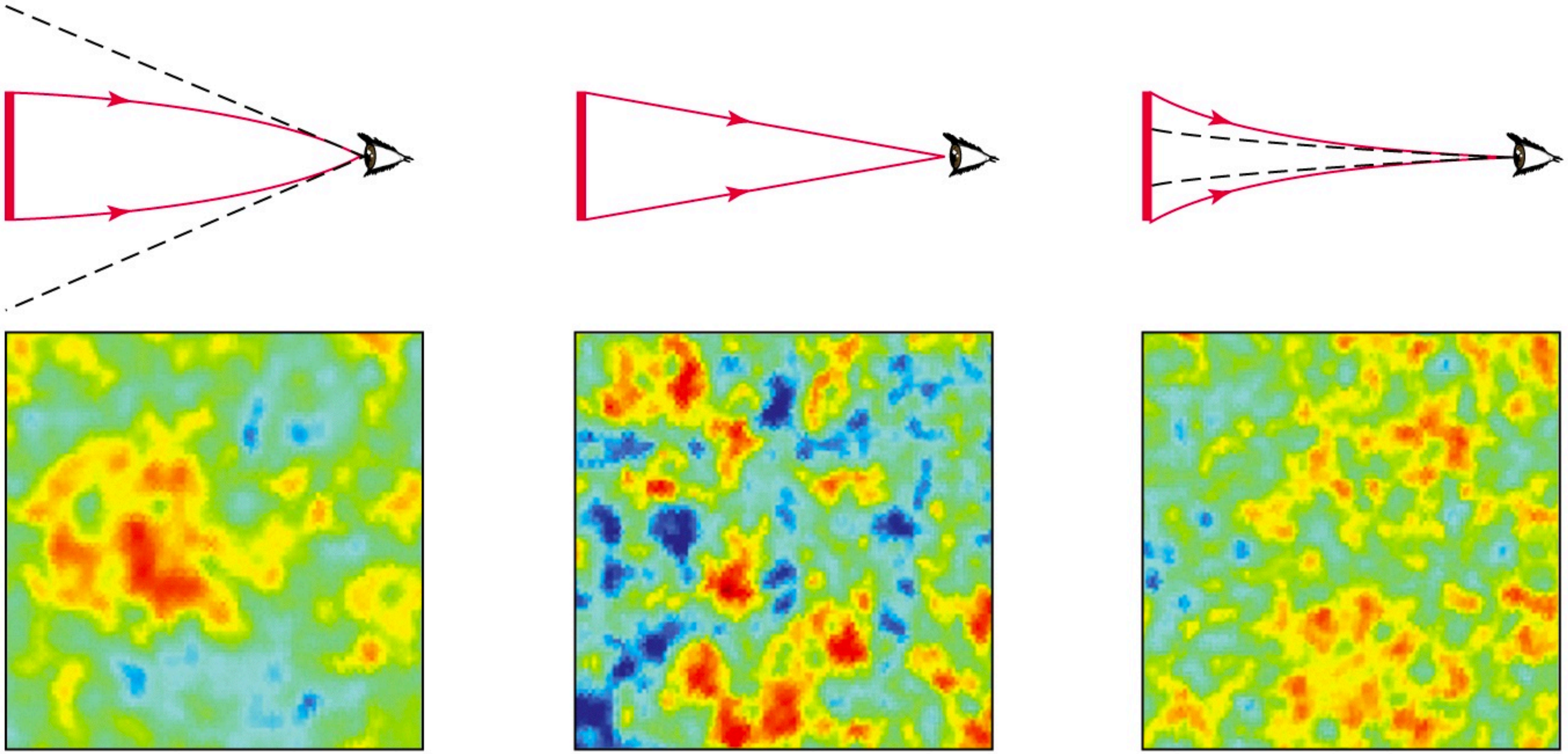
$$M = m - 5\log_{10}\left(\frac{d_L}{10\,\mathrm{pc}}\right)$$

$$M = m - 5\log_{10}(d_L/1\,\mathrm{Mpc}) - 25$$

$$m - M \simeq 43.23 - 5\log_{10} h + 5\log_{10} z + 1.086(1-q_0)z$$


HST Discovered
Ground Discovered
45
40
35
30
μ
0.5
1.0
1.5
2.0
z
Binned Gold data
high-z gray dust
Evolution ~ z
pure acceleration: q(z)=-0.5
w=-1.2, dw/dz=-0.5
w=-0.8, dw/dz=+0.5
Ω_M=1.0, Ω_Λ=0.0
~ pure deceleration: q(z)=0.5
Empty (Ω=0)
Ω_M=0.29, Ω_Λ=0.71
Δ(m-M) (mag)
0.5
0.0
-0.5
0.0
0.5
1.0
1.5
2.0
z

CMB provides a giant triangle of known size!

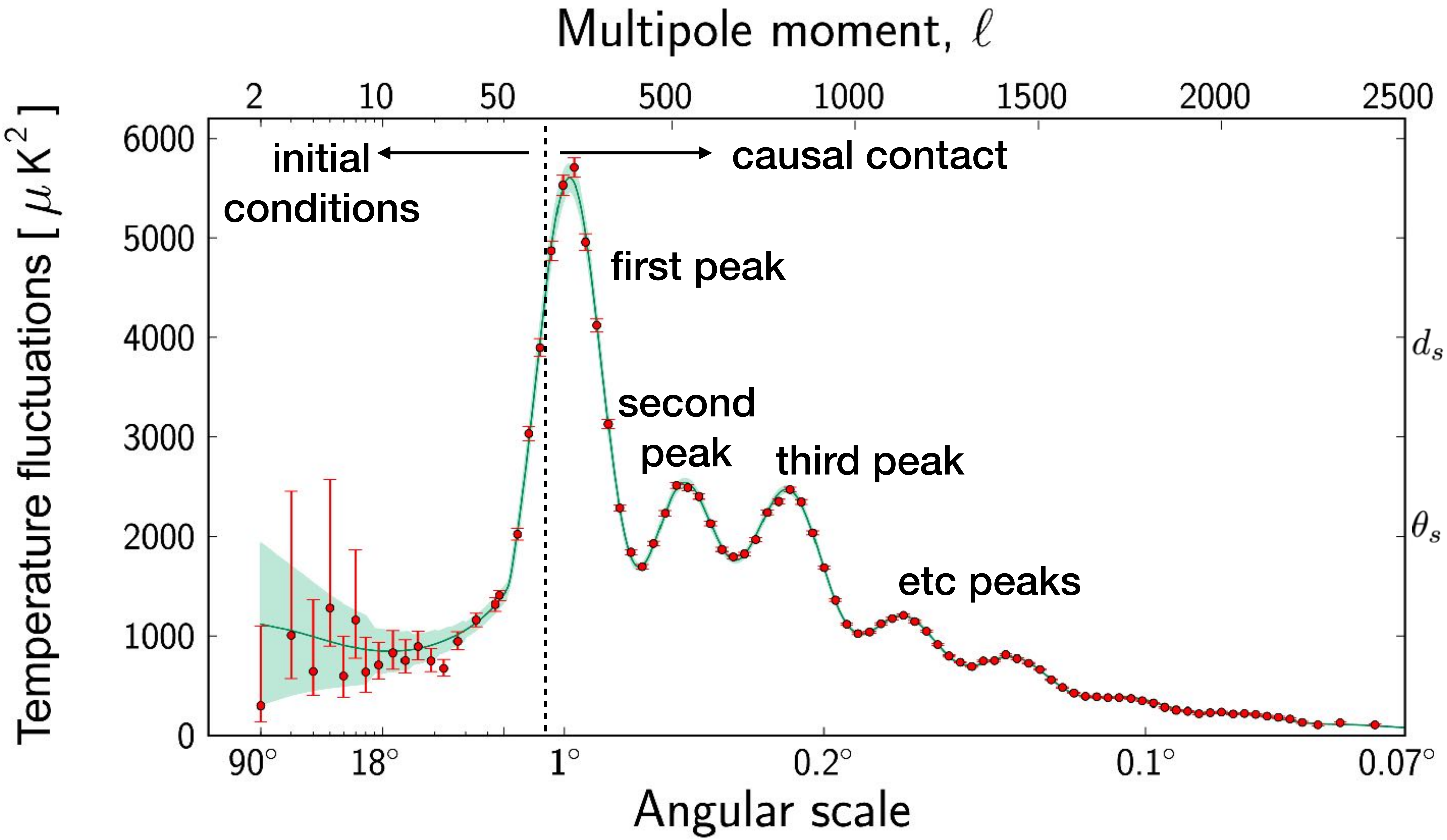


a If universe is closed, "hot spots" appear larger than actual size

b If universe is flat, "hot spots" appear actual size

c If universe is open, "hot spots" appear smaller than actual size

Acoustic peaks



$$d_s(t_{\rm ls}) = a(t_{\rm ls}) \int_0^{t_{\rm ls}} \frac{c_s(t)dt}{a(t)}$$

$$d_s(t_{\rm ls}) \approx \frac{1}{\sqrt{3}} d_{\rm hor}(t_{\rm ls}) \approx 145 \text{ kpc}$$

$$\theta_s \approx \frac{d_s(t_{\rm ls})}{d_A} \approx \frac{145 \text{ kpc}}{12.8 \text{ Mpc}} \approx 0.7^\circ$$

size scale of a DM potential well where baryon collapse reaches turnaround due to its pressure

Baryonic Matter

$$\Omega_{*,0} \lesssim 0.005$$

$$M_{\mathrm{gas},0} \approx 10 \times M_{*,0}$$

early universe measurements

$$\Omega_{\mathrm{bary},0} = 0.048 \pm 0.003$$

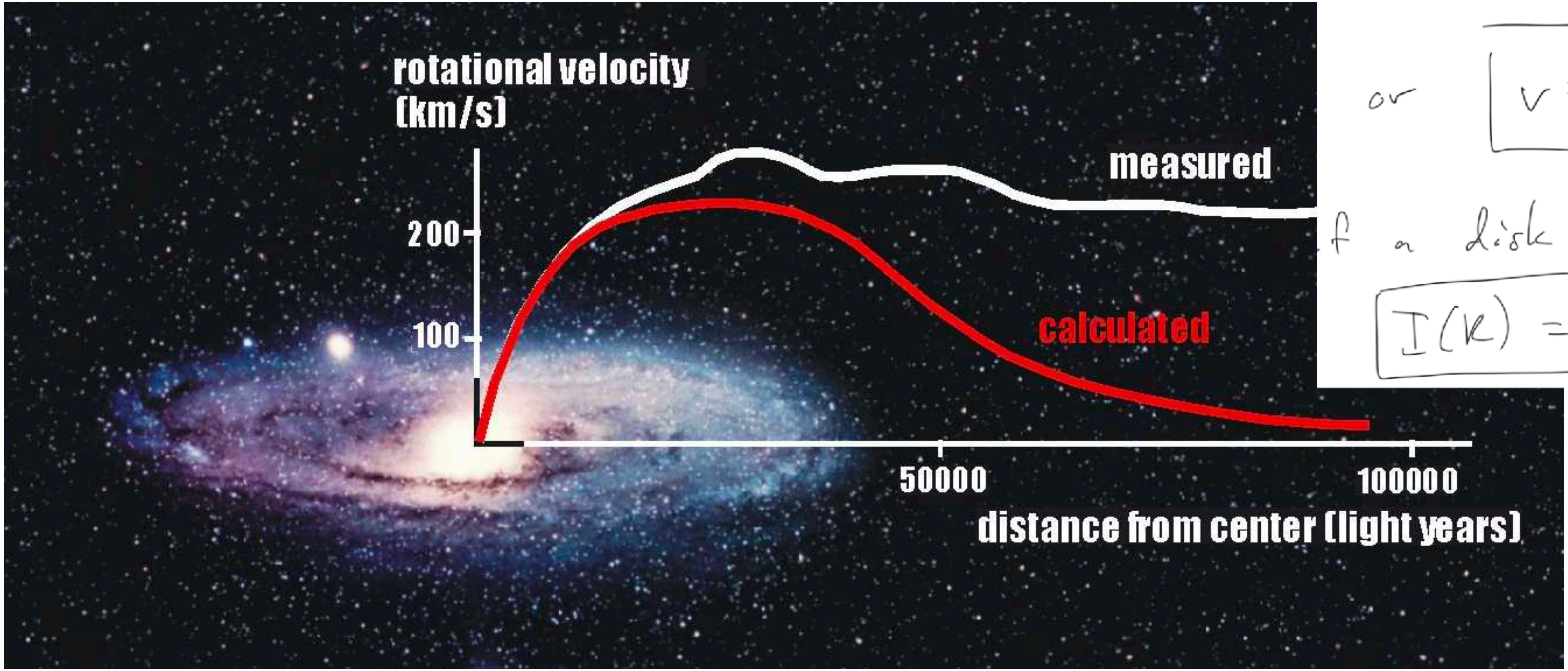
$$\Omega_{m,0} = 0.31$$

baryonic matter only 15%

By the time of the Big Bang and thereafter, normal matter is the subdominant form of matter in the universe, with some other form of matter (non-baryonic dark matter) making up the majority of non-relativistic matter in the universe

Could be primordial black holes that were made before this time (i.e., not from stars).

Dark Matter in Galaxies



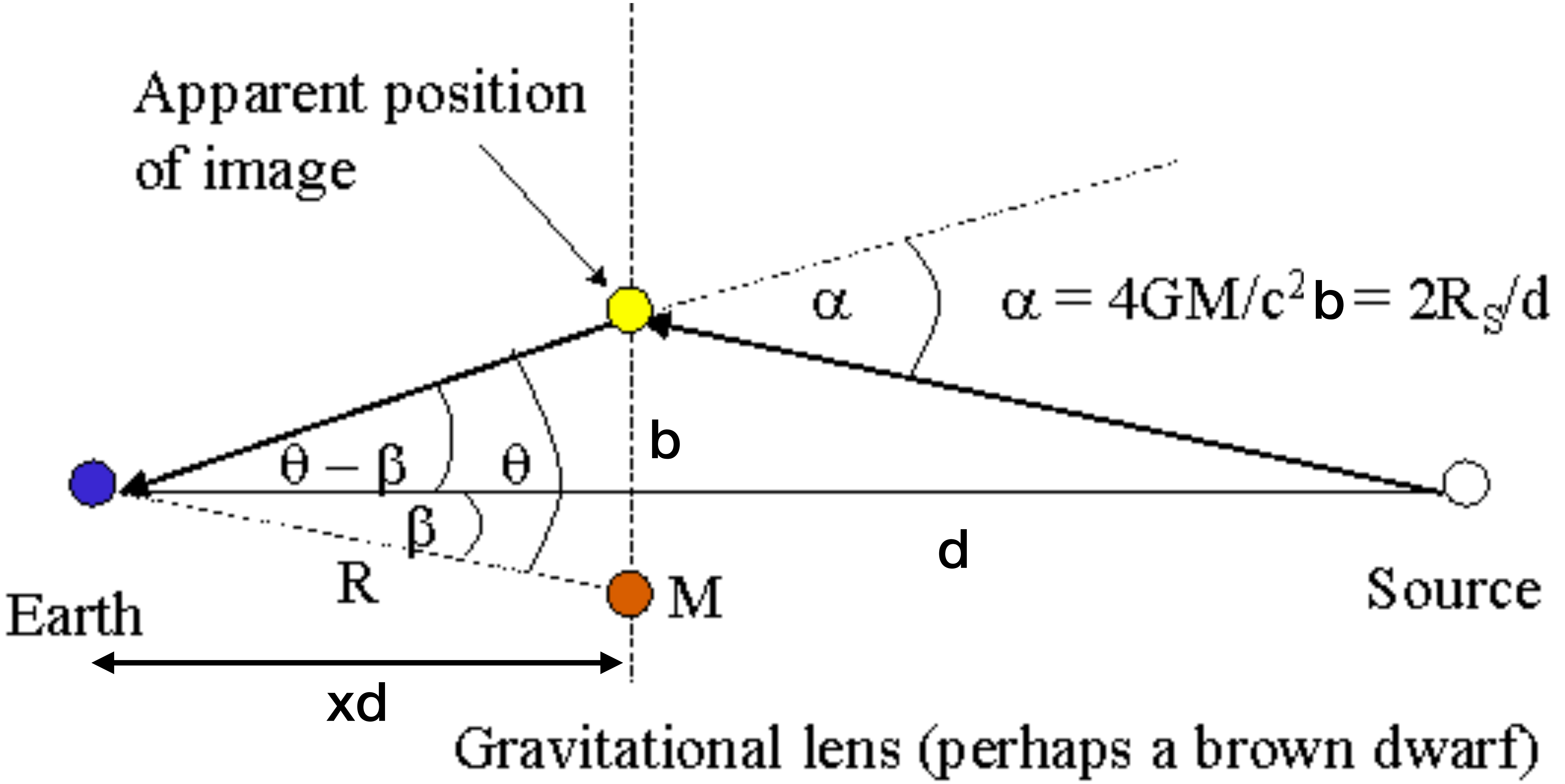
or $v = \sqrt{\frac{GM(<R)}{R}}$

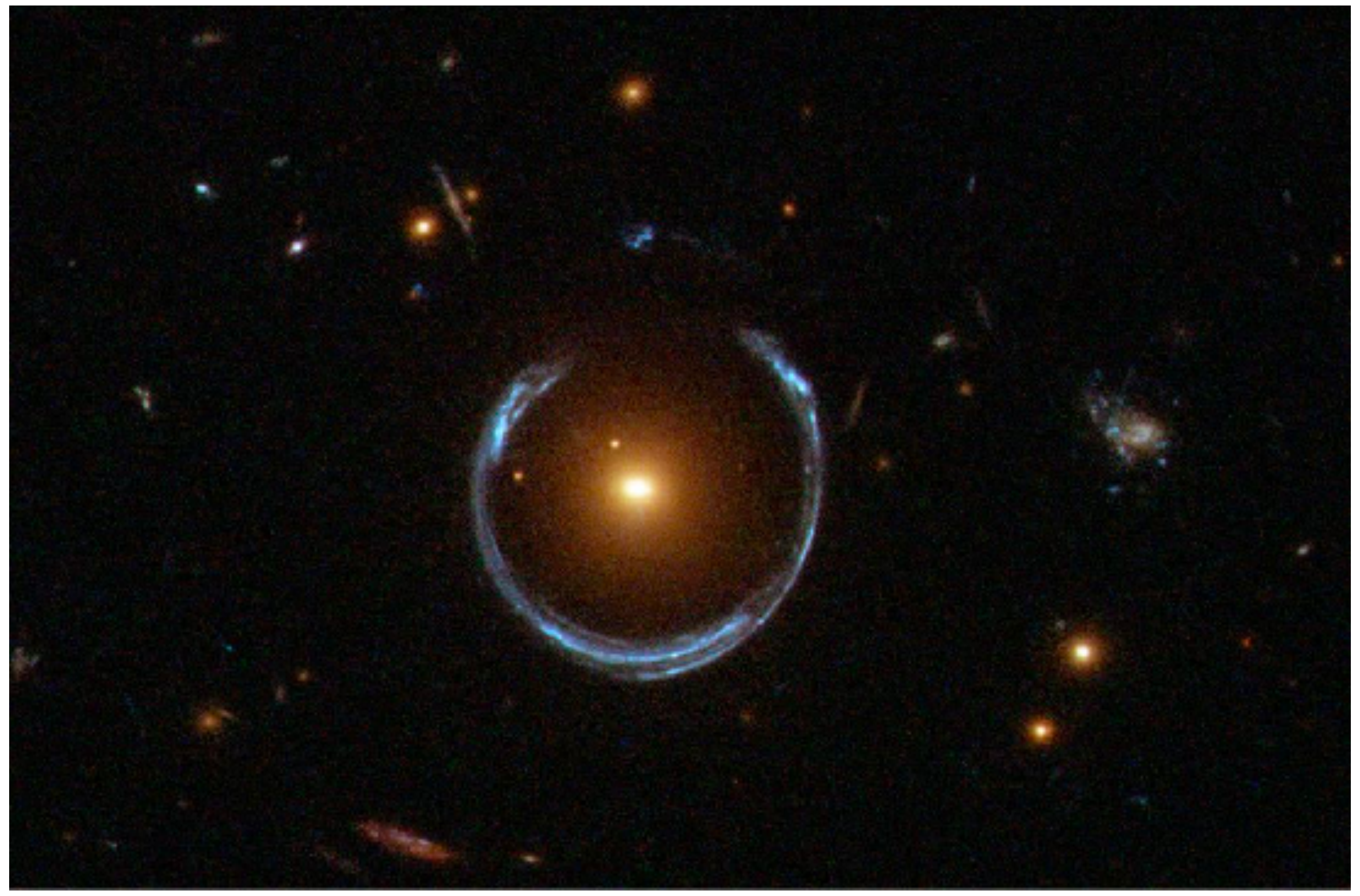
f a disk galaxy typical

$I(R) = I(0)\, e^{-R/R_s}$

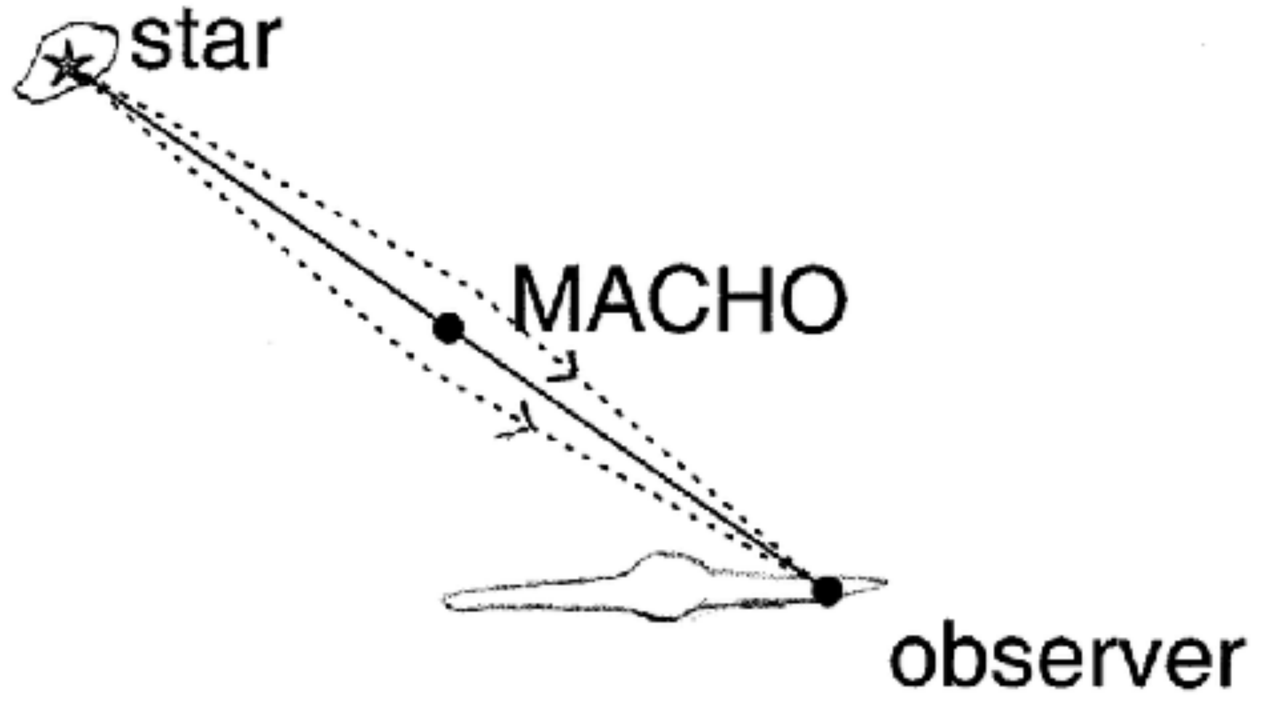
$$M(R) = \frac{v^2 R}{G} = 1.05 \times 10^{11}\ \mathrm{M}_\odot \left(\frac{v}{235\ \mathrm{km\ s^{-1}}}\right)^2 \left(\frac{R}{8.2\ \mathrm{kpc}}\right)$$

Detecting MACHOs via gravitational lensing





$$\theta_E = \left(\frac{4GM}{c^2 d} \frac{1-x}{x} \right)^{1/2}$$



R = distance to lens (of mass M)

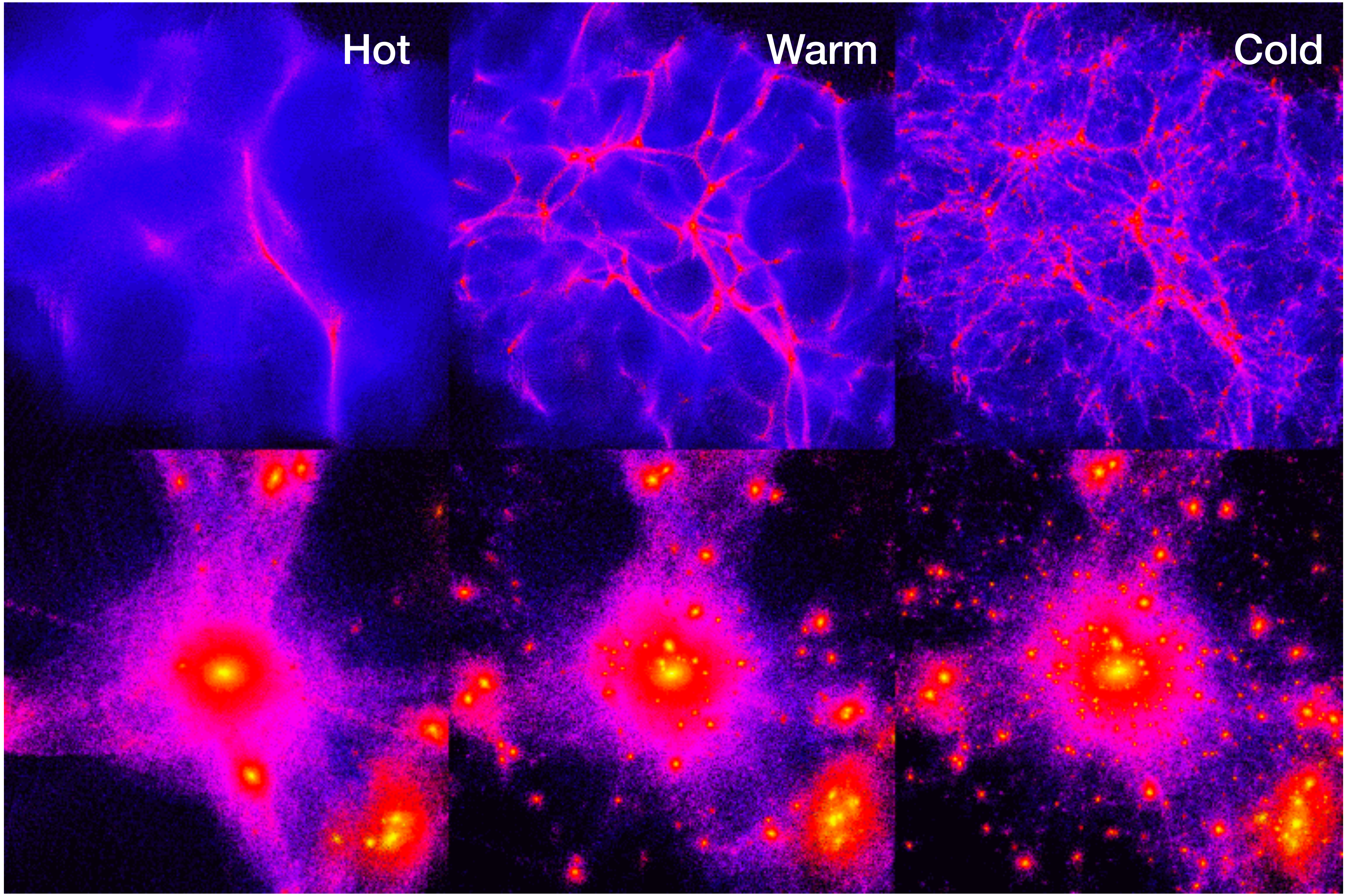
d = distance to source

b = distance between lens and image

R_S = Schwarzchild radius of lens

$$\theta_E \approx 4 \times 10^{-4} \text{ arcsec } \times \left(\frac{M}{1\ M_\odot} \right)^{1/2} \left(\frac{d}{50\ \text{kpc}} \right)^{-1/2}$$

Temperature of the Dark Matter



velocity of particles compared to the speed of light

relativistic at time of collapse (like neutrinos): hot

non-relativistic at time of collapse (like WIMPs): cold

fast motions wipe out initial overdensities on small scales: “free-streaming”

Power spectrum of density fluctuations

Power spectrum defined to be the mean squared amplitude of the Fourier components:

$$P(k) = \left\langle |\delta_{\vec{k}}|^2 \right\rangle$$

Gaussian field: each component uncorrelated and random, drawn from the Gaussian distribution

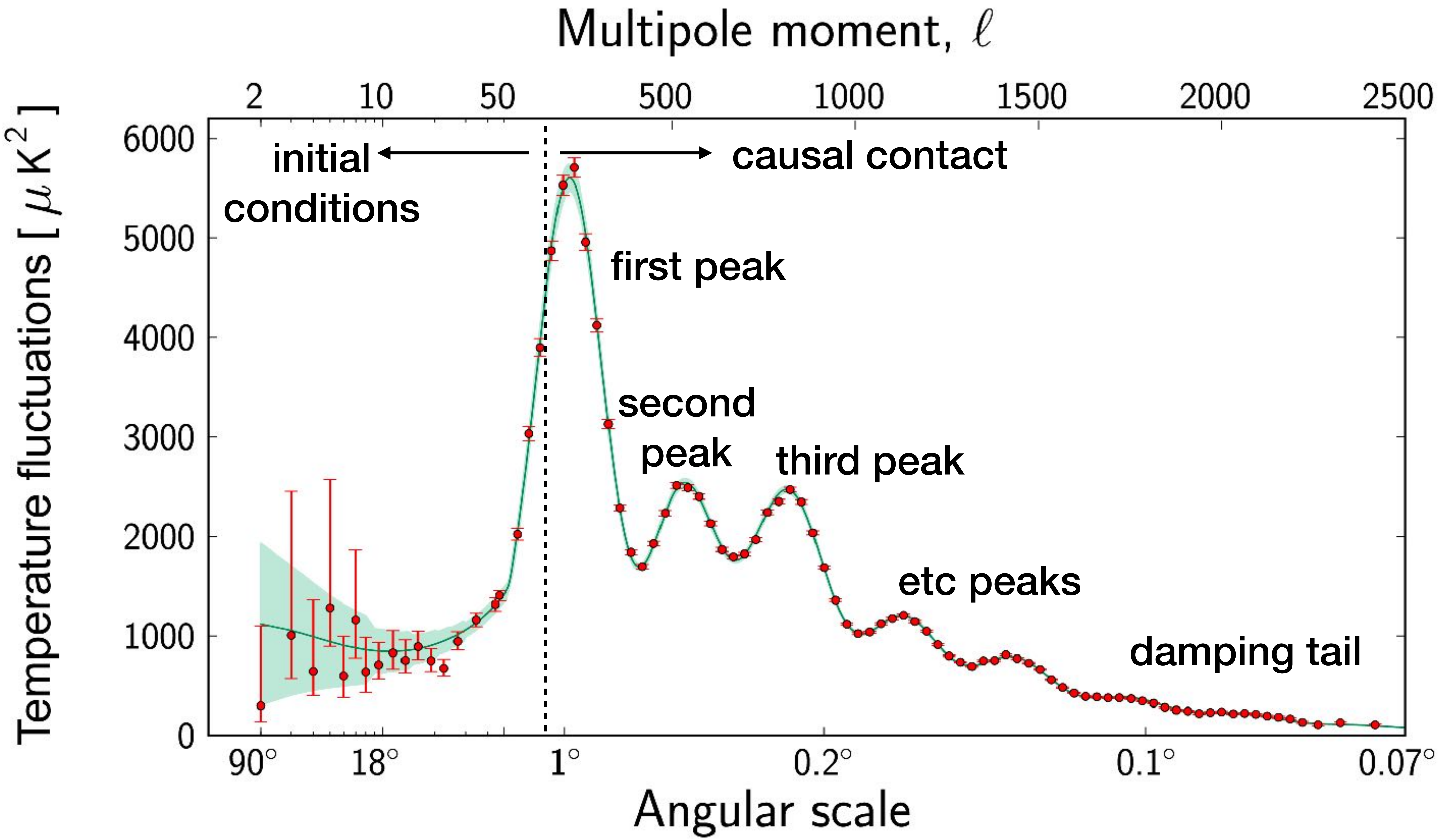
$$p(\delta) = \frac{1}{\sigma_\delta \sqrt{2\pi}} \exp\left(-\frac{\delta^2}{2\sigma_\delta}\right)$$

$$\sigma_\delta^2 = \frac{V}{2\pi^2} \int_0^\infty P(k) k^2 dk$$

Inflation predicts this (random quantum fluctuations) and a power law power spectrum (with n=1)

$$P(k) \propto k^n$$

Acoustic peaks



First peak:
spatially flat

Second peak:
existence of "dark baryons"

Third peak:
amount of dark matter

Damping tail:
photons can cross entire grav. fluct., wipes out signal

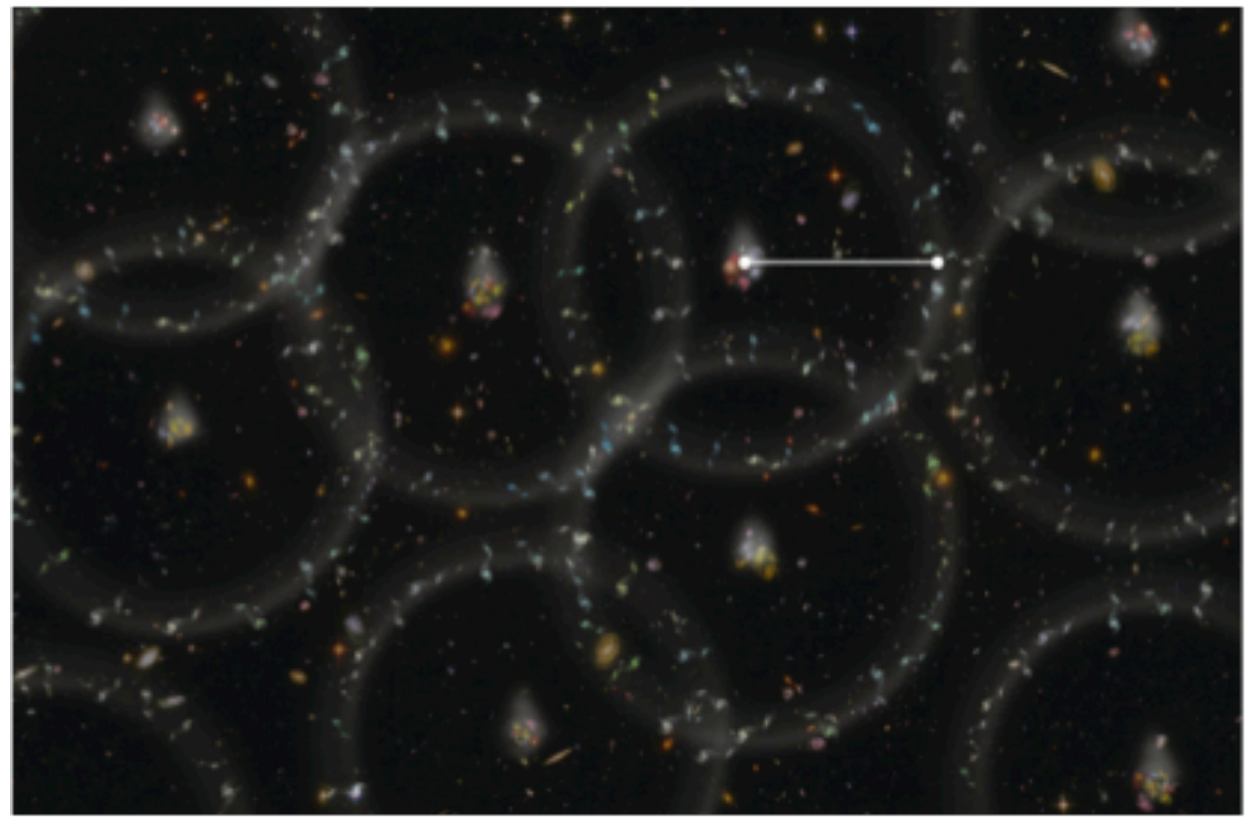
Baryon Acoustic Oscillations

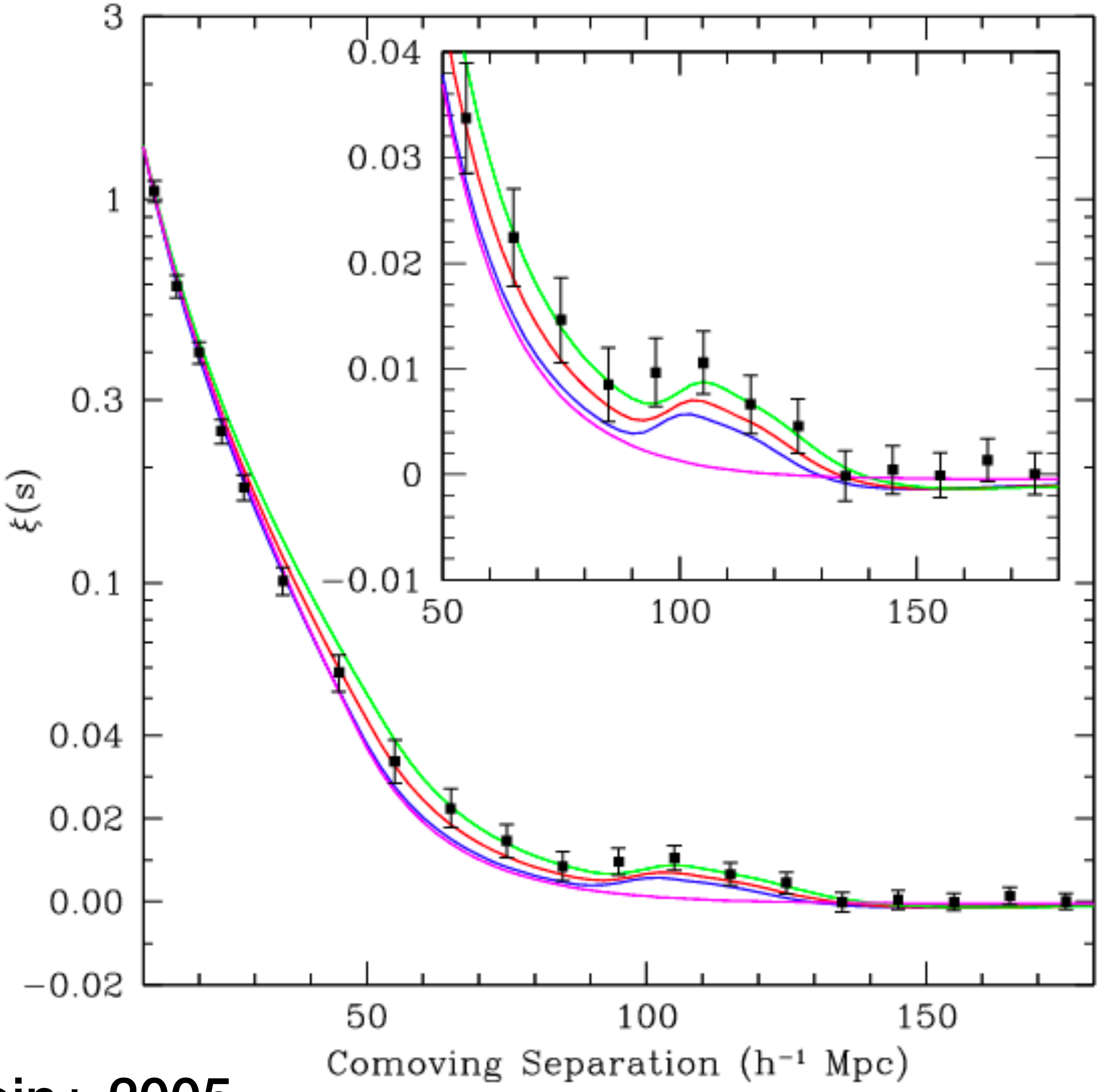
To measure, use galaxies to trace the signature of these oscillations

The number of galaxies should be correlated with each other on scales comparable to the sound horizon of the largest acoustic peaks (~150 Mpc comoving)

The number of galaxies within a given volume is

$$dN = n_{\rm gal}[1 + \xi(r)]dV$$





Eisenstein+ 2005

Open Questions

- Is the inflationary hypothesis -- which determines the "initial conditions" that control practically everything we can now observe in the universe -- generally accurate?
- If the inflationary hypothesis is generally correct, did inflation occur in such a way that the universe we now observe is only one of countless "bubble universes" that could have arisen out of the same process?
- How many forms of dark matter are really present in the universe, what is the relative percentage of each, and how has the dark matter affected the observable structure of the universe?
- What is the true cause of the accelerating expansion we seem to observe now, and is it likely to continue indefinitely into the future?
- What is the true dimensionality of spacetime, and how did the apparent three dimensions of space and one of time (along with any other "hidden" dimensions) come to be as we see them?
- Are the laws of physics, in particular the "fundamental constants", truly the same everywhere and at every time in the observable universe, or do they vary in some slight but predictable way?
- Is the topology of the observable universe truly infinite, or is it finite in such a way that we could in principle observe our own section of the universe if we could only see "far" enough?

from http://www.openquestions.com/oq-cosmo.htm#questions